Math 313/513, Spring 2021 **Martin Bies**

Homework 1

Due: Thursday, January 28 – 10:00 am EST

Conventions

- Submit your handwritten solutions in a *single* pdf-file to *Canvas*. **Indicate on each page which exercise is being solved.**
- For historic reasons, `Python` 2 and `Python` 3 are available. For reasons of consistency, **we agree to use `Python` 3 code** in `Jupyter` notebooks. Once the computations are completed, download your notebook as html-file.
- A complete solution for this assignment consists of one pdf-file with your handwritten notes and the html-file corresponding to your `Jupyter` notebook.

Problem 1: Installation and configuration of `Python` [10 points]

1. Install `Python` 3 and verify that your version is ≥ 3.0.
2. Install the packages *numpy* and *scipy* for `Python` 3.
3. Install `Jupyter` and start a notebook with `Python` 3.
4. Perform the following tasks in this `Jupyter` notebook. `Jupyter` supports headings and, in markdown mode, even LaTeX. Use this to add interpretations and explanations directly to your notebook.
   - Add a cell with the following content and execute it:

     ```
     from platform import python_version
     print(python_version())
     ```

   - Add a cell with the following content and execute it:

     ```
     print( "HelloWorld!" )
     print "HelloWorld!"
     ```

     Explain why the second line triggers an error.
   - Execute the following code in a cell. It uses the packages *numpy* and *scipy* for linear algebra:

     ```
     # import numpy and the linear algebra tools in scipy
     import numpy as np
     import scipy.linalg as la

     # create a matrix and a vector
     M = np.array([[1,2],[3,7]])
     ```

```
v = np.array([[1],[2]])

# compute the product of M and v
print( "M*v:" )
print( str( M @ v ) + "\n" )

# compute the inverse of M
print( "M^-1:" )
print( str( la.inv(M) ) + "\n" )
```

- Extend the notebook to compute and print the matrix products $M \cdot M^{-1}$ and $M^{-1}M$. Compare the computed results with your expectation. Interpret your findings.

Problem 2: Elementary operations with vectors and matrices [10 points]
Perform the following tasks without `Python`.

1. For $\vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \vec{y} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \in \mathbb{R}^2$ compute $\vec{x} + \vec{y}$ and $2 \cdot \vec{x}$.
2. Draw an image containing $\vec{x}$, $\vec{y}$, $\vec{x} + \vec{y}$ and $2 \cdot \vec{x}$.
3. Find $a, b \in \mathbb{R}$ with $a\vec{x} + b\vec{y} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$.
4. For $P = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, compute $P\vec{x}$ and $P\vec{y}$. Interpret the result.
5. For $R = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$, compute $R\vec{x}$ and $R\vec{y}$. Interpret the result.

Problem 3: Automation with `Python/Jupyter` [10 points]
Extend your `Jupyter` notebook such that it executes the tasks in problem 2 and prints the results. This includes a plot of the vectors!

Problem 4: Interesting matrices [10 points]
Find 2×2 matrices $M \in \mathbb{M}(2 \times 2, \mathbb{R})$ and vectors $\vec{b} \in \mathbb{R}^2$ with the following properties:

- $M\vec{x} = \vec{b}$ has no solution $\vec{x}$,
- $M\vec{x} = \vec{b}$ has exactly one solution $\vec{x}$,
- $M\vec{x} = \vec{b}$ has infinitely many solutions $\vec{x}$, but for at least one $\vec{x} \in \mathbb{R}^2$: $M\vec{x} \neq \vec{b}$.
- $M\vec{x} = \vec{b}$ holds true for all $\vec{x} \in \mathbb{R}^2$.

Give a geometric interpretation ("row picture") of each case.