Homework 1 – Theory

Due: Thursday, January 20 – 10:00 am EST

Problem 1T: Elementary operations with vectors and matrices [20 points]

1. For $\vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \vec{y} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \in \mathbb{R}^2$, draw an image of $\vec{x}$, $\vec{y}$, $\vec{x}+\vec{y}$, $2 \cdot \vec{x}$.
2. Find $a, b \in \mathbb{R}$ with $a\vec{x} + b\vec{y} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$.
3. For $P = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, compute $P\vec{x}$ and $P\vec{y}$. Interpret the result.
4. For $R = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, compute $R\vec{x}$ and $R\vec{y}$. Interpret the result.

Problem 2T: Interesting matrices [20 points]

Find 2×2 matrices $M \in \mathbb{M}(2 \times 2, \mathbb{R})$ and vectors $\vec{b} \in \mathbb{R}^2$ with the following properties:

- $M\vec{x} = \vec{b}$ has no solution $\vec{x}$,
- $M\vec{x} = \vec{b}$ has exactly one solution $\vec{x}$,
- $M\vec{x} = \vec{b}$ holds true for all $\vec{x} \in \mathbb{R}^2$.
- **For Math 513**:
  $M\vec{x} = \vec{b}$ has infinitely many solutions $\vec{x}$, but for at least one $\vec{x} \in \mathbb{R}^2$: $M\vec{x} \neq \vec{b}$.

Give a geometric interpretation ("row picture") of each case.