Math 313/513, Spring 2022 **Martin Bies**

Homework 6 – Coding

Due: Thursday, March 24 – 10:00 EST

Problem 1C: Least square approximation in `Python` [20 Points]

1. Write a function `LineFit` which accepts $[(t_1, b_1), (t_2, b_2), \ldots, (t_n, b_n)]$, fits a line to this data, plot this line and the data points (t_i, b_i).
   Hint: Use orthogonal projection as discussed in class.
2. Similarly, write a function `ParaFit`, which fits a parabola $C + Dt + Et^2$ to $[(t_1, b_1), (t_2, b_2), \ldots, (t_n, b_n)]$, plots this parabola and the data points.
3. Apply `LineFit` and `ParaFit` to $[(1, 2), (2, 2), (3, 5), (4, 3), (4.5, 8)]$. By looking at the plots, does the line or the parabola describe the data better?
4. Expand your functions by a criterion for the quality of the fit. Justify your criterion and use it to tell if the line or parabola fits the data better.
5. **Math 513:** Compare your line fit with the linear regression fit in *scikit-learn*.

Problem 2C: Fourier series in `Python` [20 Points]

1. Write a function r:
   - Input: $x \in [0, 2\pi]$
   - Output: 1 if $0 \leq x \leq \pi$ and 0 otherwise.
2. Write a function `FourierTransform`:
   - Input: A function f (such as r from part 1) and $d \in \mathbb{Z}_{\geq 0}$.
   - Output: Scatter plot of a_0, a_k, b_k $(1 \leq k \leq d)$ in different colours.
3. Apply `FourierTransform` to r for $d = 10$. Qualitatively, describe the plot.
4. Write a function `FourierSeries`:
   - Input: A function f (such as r from part 1) and $d \in \mathbb{Z}_{\geq 0}$.
   - Output:
     - A `Python` function $F\colon [0, 2\pi] \to \mathbb{R}\,,\ x \mapsto F(x)$ with

$$F(x) = a_0 + \sum_{k=1}^{d} a_k \cdot \cos(kx) + \sum_{k=1}^{d} b_k \cdot \sin(kx)\,. \qquad (1)$$

     - Plot of $f(x)$ and $F(x)$ for at least 500 position x with $0 \leq x \leq 2\pi$.
5. Apply `FourierSeries` to r for $d = 3, 10, 50$ and describe the plots.

6. For $d = 3, 10$, apply `FourierTransform` and `FourierSeries` to

$$g\colon [0, 2\pi] \to \mathbb{R}\,,\ x \mapsto e^{-(x-\pi)^2}\,. \tag{2}$$

7. **Math 513:** Why does `FourierSeries` converge much quicker for g than for r?