Homework 6 – Theory

Due: Thursday, March 24 – 10:00 EST

Problem 1T: Hesse normal form [10 Points]

1. Be $\vec{a}, \vec{b} \in \mathbb{R}^3 \setminus \vec{0}$ two linearly independent vectors. For $\vec{x}_0 \in \mathbb{R}^3$ consider

   $$S\left(\vec{x}_0\right) = \left\{ \mu\vec{a} + \nu\vec{b} + \vec{x}_0 \,|\, \mu, \nu \in \mathbb{R} \right\} \subseteq \mathbb{R}^3 \,. \tag{1}$$

   Show that there exist $\vec{n} \in \mathbb{R}^3 \setminus \vec{0}$ and $d \in \mathbb{R}$ such that $\vec{n}^T\vec{n} = 1$ and

   $$\vec{x} \in S\left(\vec{x}_0\right) \qquad \Leftrightarrow \qquad \langle \vec{n}, \vec{x} \rangle_{\text{std}} - d = 0 \,. \tag{2}$$

2. Give a geometric interpretation of $\vec{n} \in \mathbb{R}^3 \setminus \vec{0}$ and $|d| \in \mathbb{R}$.
3. Assume $\vec{x}_0 \neq \vec{0}$: Are $\vec{n} \in \mathbb{R}^3$ and $d \in \mathbb{R}$ unique? If not, name conditions under which they are.
4. **Bonus (for 313 and 513 students)**
   Assume $\vec{x}_0 = \vec{0}$. Is there a condition which fixes $\vec{n} \in \mathbb{R}^3$ and $d \in \mathbb{R}$ uniquely?

Problem 2T: QR-decomposition [10 Points]

1. Explain why QR-decompositions are useful.
2. Compute a QR-decomposition for $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \in \mathbb{M}(3 \times 3, \mathbb{R})$.
3. Is the QR-decomposition unique? If not, what conditions make it unique?