Math 313/513, Spring 2022 **Martin Bies**

Homework 7 – Coding

Due: Thursday, March 31 – 10:00 EST

Problem 1C: Basic diagonalization in `Python` [20 Points]

1. Use `numpy` to write a `Python` function `BasicDiag` which realizes the following algorithm:
   - Input: $A \in \mathbb{M}(n \times n, \mathbb{R})$,
   - Output: $A_{\mathcal{B}_1\mathcal{B}_1}$, $T_{\mathcal{B}_1\mathcal{B}_2}$.

   The matrix $A_{\mathcal{B}_1\mathcal{B}_1}$ is to be computed by the following algorithm:

   a) Check that the input matrix A is a square matrix.

   b) The zeros of $\mathrm{ch}_A(\lambda) = \det(A - \lambda I) \in \mathbb{R}[\lambda]$ are known as *eigenvalues* of A. For deep mathematical reasons, they are considered as complex numbers. Use the build in functions in `numpy` to compute the eigenvalues of A.

   c) Proceed if there are exactly n *distinct* and *real* eigenvalues $\lambda_1, \ldots, \lambda_n \in \mathbb{R}$. Otherwise, raise a warning.

   d) For each λ_i compute a so-called *eigenvector* $\vec{v}_i \in \mathbb{R}^n$, that is $A\vec{v}_i = \lambda_i \cdot \vec{v}_i$.

   e) Proceed if $\mathcal{B}_1 = \{\vec{v}_1, \ldots, \vec{v}_n\}$ is a basis of $\mathbb{R}^n$. Otherwise, raise an error.

   f) Let $\mathcal{B}_2 = \{\vec{e}_1, \ldots, \vec{e}_n\}$ be the standard basis of $\mathbb{R}^n$. Construct the base change matrix $T_{\mathcal{B}_2\mathcal{B}_1}$ and compute $A_{\mathcal{B}_1\mathcal{B}_1} = T_{\mathcal{B}_1\mathcal{B}_2} A_{\mathcal{B}_2\mathcal{B}_2} T_{\mathcal{B}_2\mathcal{B}_1}$.

2. Apply `BasicDiag` to $A = I_3$. "Too few eigenvalues" should be triggered.

3. Apply `BasicDiag` to $A \in \mathbb{M}(3 \times 3, \mathbb{R})$ with

$$A = \begin{bmatrix} -2 & -2 & -2 \\ -2 & 1 & -5 \\ -2 & -5 & 1 \end{bmatrix} . \tag{1}$$

   You should find a result *equivalent* to your answer in problem 3T.

4. Apply `BasicDiag` to solve $A\vec{x} = \vec{b}$ with

$$A = \begin{bmatrix} -1 & 3 & 1 \\ 2 & 3 & 3 \\ 3 & 5 & 6 \end{bmatrix} \in \mathbb{M}(3 \times 3, \mathbb{R}) , \qquad \vec{b} = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} . \tag{2}$$

   Compare with your answer to problem 1T.