CAPPresentation-Category

The CAP presentation category based on a projective category (e.g. CAPCategory-OfProjectiveGradedModules)

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Contents

Chapter 1

Presentation Category

1.1 Constructor

1.1.1 PresentationCategory (for IsCapCategory)

▷ `PresentationCategory(`*P*`)` (attribute)

Returns: a category

The argument is a so-called proj-category *P*. That is an additive category with weak lifts, weak kernels and a rigid, symmmetric, closed monoidal structure. The output is then the presentation category over this proj-category. Given that *P* is a strict monoidal category, or that direct sums with the zero object of *P* lead to identical objects, can simplify the structure of the presentation category (in that unitors become identities - not sure about the associators yet!) This could in principle be used, but is not used as of now. FIX ME FIX ME FIX ME).

Chapter 2

Objects

2.1 GAP Category

2.1.1 IsCAPPresentationCategoryObject (for IsCapCategoryObject)

▷ `IsCAPPresentationCategoryObject(`*object*`)` (filter)

Returns: `true` or `false`

The GAP category of objects in the presentation category over a proj-category *P*.

2.2 Constructors

2.2.1 CAPPresentationCategoryObject (for IsCapCategoryMorphism)

▷ `CAPPresentationCategoryObject(`*m*`)` (operation)

Returns: a CAPCategoryObject

The argument is a CAPCategoryMorphism *m* in a Proj-category. This morphism is then considered as an object in the presentation category.

For morphisms we support the possibility to turn off consistency checks on the input data. For the constructor of objects this is not possible. This is because the only check is if the morphism is a morphism in a Proj-category. This in turn is checked by constructing the presentation category of the category in which the morphism is defined. The latter in turn is an attribute of a CapCategory and thus is constructed only once. In this sense the given constructor is already optimal.

2.3 Attributes

2.3.1 UnderlyingMorphism (for IsCAPPresentationCategoryObject)

▷ `UnderlyingMorphism(`*A*`)` (attribute)

Returns: a CAPCategoryMorphism

The argument is an object *A* of the presentation category over a proj-category *P*. Every such object is encoded by a morphism in the underlying proj-category. The output is such a morphism in the proj-category.

2.4 Printing all information about an object

2.4.1 FullInformation (for IsCAPPresentationCategoryObject)

▷ `FullInformation(A)` (operation)

Returns: detailed information about the object A

The argument is an object A in the presentation category. For such an object it can be tedious to work out all the details of the underlying defining morphism (such a source, ranges and the mapping matrix). To encompass this shortcoming, this method produces all this information with just a single command.

2.5 Hom-Embedding

2.5.1 INTERNAL_HOM_EMBEDDING_IN_TENSOR_PRODUCT (for IsCAPPresentationCategoryObject, IsCAPPresentationCategoryObject)

▷ `INTERNAL_HOM_EMBEDDING_IN_TENSOR_PRODUCT(a, b)` (operation)

Returns: a morphism

The arguments are two objects `a` and `b` of the presentation category. Given that a is represented as $\alpha: R_A \to A$, we have the exact sequence $0 \to Hom(a,b) \to A^\vee \otimes b \to R_A^\vee \otimes b$. The latter map is given by $\alpha^\vee \otimes 1_b$. The kernel of this map is the Hom-embedding computed by this map.

Chapter 3

Morphisms

3.1 The GAP category for morphisms

3.1.1 IsCAPPresentationCategoryMorphism (for IsCapCategoryMorphism)

▷ `IsCAPPresentationCategoryMorphism(`*object*`)` (filter)

Returns: `true` or `false`

The GAP category of morphisms in the presentation category over a proj-category P.

3.2 Constructors

3.2.1 CAPPresentationCategoryMorphism (for IsCAPPresentationCategoryObject, IsCapCategoryMorphism, IsCAPPresentationCategoryObject)

▷ `CAPPresentationCategoryMorphism(`*m1, a, m2*`)` (operation)

Returns: a CAPCategoryMorphism

The arguments are an object m_1 in the presentation category, a morphism a in the Underlying proj-category and a second object m_2 in the presentation category. The objects m_1 and m_2 are induced from morphisms in the underlying proj-category. Let us therefore write $m_1{:}A \to B$ and $m_2{:}C \to D$. Then this data must be such that the source of a equals B and the range of a equals D. Moreover there must exists a sourcelift, i.e. a morphism $b{:}A \to C$ such that the diagram formed from m_1, m_2, a and b is commutative. Given that all of this is satisfied, the output is the corresponding morphism in the presentation category.

3.2.2 CAPPresentationCategoryMorphism (for IsCAPPresentationCategoryObject, IsCapCategoryMorphism, IsCAPPresentationCategoryObject, IsBool)

▷ `CAPPresentationCategoryMorphism(`*m1, a, m2*`)` (operation)

Returns: a CAPCategoryMorphism

The arguments are an object m_1 in the presentation category, a morphism a in the Underlying proj-category, a second object m_2 in the presentation category and a boolean *checks_wished*. If *checks_wished = true*, this constructor behaves just like the one above, if *checks_wished = false*, not consistency checks on the given input is performed. Therefore this latter option could be useful for high-performance-applications.

3.3 Attributes

3.3.1 UnderlyingMorphism (for IsCAPPresentationCategoryMorphism)

▷ `UnderlyingMorphism(m)` (attribute)

Returns: a CAPCategoryMorphism

The argument is a morphism m in the presentation category, and the output the underlying morphism in the proj-category.

3.3.2 SourceLiftMorphism (for IsCAPPresentationCategoryMorphism)

▷ `SourceLiftMorphism(m)` (attribute)

Returns: a CAPCategoryMorphism

The argument is a morphism m in the presentation category. Then source and range of m are objects in the presentation category. In terms of the underlying proj-category we can therefore represent the source of m as $s: A \to B$ and its range as $t: C \to D$. Moreover m is induced from a morphism $\tilde{m}: B \to D$. That said, the output is a morphism $\mu: A \to C$ such that the diagram formed from s, $\tilde{m}$, μ and t is commutative.

3.4 Printing all information about a morphism

3.4.1 FullInformation (for IsCAPPresentationCategoryMorphism)

▷ `FullInformation(m)` (operation)

Returns: detailed information about the morphism

The argument is a morphism m in the presentation category. For such a morphisms it can be tedious to work out the source and ranges in detail with the ordinary display and view methods. To encompass this shortcoming, this method produces all this information with just a single command.

Chapter 4

Presentation Category Functors

4.1 Functors

4.1.1 EmbeddingOfProjCategory (for IsCapCategory)

▷ `EmbeddingOfProjCategory(`*`category`*`)` (attribute)

Returns: a functor

The argument is a Proj-category *`category`*. The output is the functor that embeds *`category`* into its presentation category.

Chapter 5

Convenience methods

5.1 Powers of objects and morphisms

5.1.1 * (for IsCAPPresentationCategoryObject, IsCAPPresentationCategoryObject)

▷ `\*(arg1, arg2)` (operation)

5.1.2 \^ (for IsCAPPresentationCategoryObject, IsInt)

▷ `\^(arg1, arg2)` (operation)

5.1.3 * (for IsCAPPresentationCategoryMorphism, IsCAPPresentationCategoryMorphism)

▷ `\*(arg1, arg2)` (operation)

5.1.4 \^ (for IsCAPPresentationCategoryMorphism, IsInt)

▷ `\^(arg1, arg2)` (operation)

Chapter 6

Examples and tests

To demonstrate the functionality of the presentation category, we construct objects and morphisms in the presentation category over the category of projective graded modules and then perform various categorical constructions with such objects and morphisms.

6.1 Constructors of objects

We first construct objects of the presentation category over the category of projective graded left S-modules, where S is the Coxring of $\mathbb{P}^1 \times \mathbb{P}^1$.

Example

```
gap> Q := HomalgFieldOfRationalsInSingular();
Q
gap> S := GradedRing( Q * "x_1, x_2, x_3, x_4" );
Q[x_1,x_2,x_3,x_4]
(weights: yet unset)
gap> SetWeightsOfIndeterminates( S, [[1,0],[1,0],[0,1],[0,1]] );

gap> Q1 := CAPCategoryOfProjectiveGradedLeftModulesObject( [ [[1,0],1] ], S );
<A projective graded left module of rank 1>
gap> Q2 := CAPCategoryOfProjectiveGradedLeftModulesObject( [ [[0,0],2] ], S );
<A projective graded left module of rank 2>
gap> Q3 := CAPCategoryOfProjectiveGradedLeftModulesObject( [ [[0,0],2] ], S );
<A projective graded left module of rank 2>
gap> Q4 := CAPCategoryOfProjectiveGradedLeftModulesObject( [ [[1,0],2] ], S );
<A projective graded left module of rank 2>
gap> m1 := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>       Q1, HomalgMatrix( [["x_1","x_2"]], S ) ,Q2 );
<A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [0, 1 ] ])>
gap> m2 := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>       Q2, HomalgMatrix( [[1,0],[0,1]], S ) ,Q3 );
<A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [0, 1 ] ])>
gap> m3 := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>       Q4, HomalgMatrix( [["x_1","x_2"], ["x_1","x_2"]], S ) ,Q3 );
<A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [0, 1 ] ])>
gap> left_category := CapCategory( Q1 );
```

```
CAP category of projective graded left modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
gap> presentation1l := CAPPresentationCategoryObject( m1 );
<An object of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> presentation2l := CAPPresentationCategoryObject( m3 );
<An object of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
```

Next we repeat these steps for right modules instead.

Example

```
gap> P1 := CAPCategoryOfProjectiveGradedRightModulesObject( [ [[1,0],1] ], S );
<A projective graded right module of rank 1>
gap> P2 := CAPCategoryOfProjectiveGradedRightModulesObject( [ [[0,0],2] ], S );
<A projective graded right module of rank 2>
gap> P3 := CAPCategoryOfProjectiveGradedRightModulesObject( [ [[0,0],2] ], S );
<A projective graded right module of rank 2>
gap> P4 := CAPCategoryOfProjectiveGradedRightModulesObject( [ [[1,0],2] ], S );
<A projective graded right module of rank 2>
gap> n1 := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>       P1, HomalgMatrix( [["x_1"],["x_2"]], S ) ,P2 );
<A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [0, 1 ] ])>
gap> n2 := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>       P2, HomalgMatrix( [[1,0],[0,1]], S ) ,P3 );
<A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [0, 1 ] ])>
gap> n3 := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>       P4, HomalgMatrix( [["x_1","x_1"], ["x_2","x_2"]], S ) ,P3 );
<A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [0, 1 ] ])>
gap> right_category := CapCategory( P1 );
CAP category of projective graded right modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
gap> presentation1r := CAPPresentationCategoryObject( n1 );
<An object of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> presentation2r := CAPPresentationCategoryObject( n3 );
<An object of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
```

6.2 All information about an object

To see all the data that defines an object in the presentation category, the command 'FullInformation' can be used.

Example

```
gap> FullInformation( presentation1l );
```

```
================================================================================

A projective graded left module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
of rank 1 and degrees:
[ [ ( 1, 0 ), 1 ] ]

A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
x_1,x_2
(over a graded ring)

A projective graded left module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
of rank 2 and degrees:
[ [ 0, 2 ] ]

================================================================================
```

6.3 Morphisms in the presentation category

Example

```
gap> presentation_morphism_l := CAPPresentationCategoryMorphism(
>                           presentation1l, m2, presentation2l );
<A morphism of the presentation category over the CAP category of projective graded
left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> IsWellDefinedForMorphisms( presentation_morphism_l );
true
gap> FullInformation( presentation_morphism_l );

================================================================================

Source:
-------
A projective graded left module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
of rank 1 and degrees:
[ [ ( 1, 0 ), 1 ] ]

A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [0, 1 ] ])
with matrix:
x_1,x_2
(over a graded ring)

A projective graded left module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
of rank 2 and degrees:
[ [ 0, 2 ] ]
```

```
------------------------------------------------------------------------------

Mapping matrix:
---------------
A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [0, 1 ] ])
with matrix:
1,0,
0,1
(over a graded ring)

------------------------------------------------------------------------------

Range:
------
A projective graded left module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
of rank 2 and degrees:
[ [ ( 1, 0 ), 2 ] ]

A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [0, 1 ] ])
with matrix:
x_1,x_2,
x_1,x_2
(over a graded ring)

A projective graded left module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
of rank 2 and degrees:
[ [ 0, 2 ] ]

==============================================================================

gap> Display( SourceLiftMorphism( presentation_morphism_l ) );
A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [0, 1 ] ])
with matrix:
1,0
(over a graded ring)
gap> FullInformation( ColiftAlongEpimorphism( presentation_morphism_l,
>                                             presentation_morphism_l ) );

==============================================================================

Source:
-------
A projective graded left module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) of rank 2 and degrees:
[ [ ( 1, 0 ), 2 ] ]
```

```
A morphism in the category of projective graded left modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
x_1,x_2,
x_1,x_2
(over a graded ring)

A projective graded left module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) of rank 2 and degrees:
[ [ 0, 2 ] ]

------------------------------------------------------------------------------

Mapping matrix:
---------------
A morphism in the category of projective graded left modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
1,0,
0,1
(over a graded ring)

------------------------------------------------------------------------------

Range:
------
A projective graded left module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) of rank 2 and degrees:
[ [ ( 1, 0 ), 2 ] ]

A morphism in the category of projective graded left modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
x_1,x_2,
x_1,x_2
(over a graded ring)

A projective graded left module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) of rank 2 and degrees:
[ [ 0, 2 ] ]

==============================================================================

gap> presentation_morphism_r := CAPPresentationCategoryMorphism(
>                           presentation1r, n2, presentation2r );
<A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> IsWellDefinedForMorphisms( presentation_morphism_r );
true
gap> FullInformation( presentation_morphism_r );

==============================================================================

Source:
```

```
-------
A projective graded right module over Q[x_1,x_2,x_3,x_4] (with weights\
 [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) of rank 1 and degrees:
[ [ ( 1, 0 ), 1 ] ]

A morphism in the category of projective graded right modules over Q[x\
_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ]\
 ]) with matrix:
x_1,
x_2
(over a graded ring)

A projective graded right module over Q[x_1,x_2,x_3,x_4] (with weights\
 [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) of rank 2 and degrees:
[ [ 0, 2 ] ]

--------------------------------------------------------------------------------

Mapping matrix:
---------------
A morphism in the category of projective graded right modules over Q[x\
_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ]\
 ]) with matrix:
1,0,
0,1
(over a graded ring)

--------------------------------------------------------------------------------

Range:
------
A projective graded right module over Q[x_1,x_2,x_3,x_4] (with weights\
 [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) of rank 2 and degrees:
[ [ ( 1, 0 ), 2 ] ]

A morphism in the category of projective graded right modules over Q[x\
_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ]\
 ]) with matrix:
x_1,x_1,
x_2,x_2
(over a graded ring)

A projective graded right module over Q[x_1,x_2,x_3,x_4] (with weights\
 [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) of rank 2 and degrees:
[ [ 0, 2 ] ]

================================================================================

gap> Display( SourceLiftMorphism( presentation_morphism_r ) );
A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
```

```
1,
0
(over a graded ring)
gap> FullInformation( ColiftAlongEpimorphism( presentation_morphism_r,
>                                             presentation_morphism_r ) );

==========================================================================

Source:
-------
A projective graded right module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) of rank 2 and degrees:
[ [ ( 1, 0 ), 2 ] ]

A morphism in the category of projective graded right modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
x_1,x_1,
x_2,x_2
(over a graded ring)

A projective graded right module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) of rank 2 and degrees:
[ [ 0, 2 ] ]

--------------------------------------------------------------------------

Mapping matrix:
---------------
A morphism in the category of projective graded right modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
1,0,
0,1
(over a graded ring)

--------------------------------------------------------------------------

Range:
------
A projective graded right module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) of rank 2 and degrees:
[ [ ( 1, 0 ), 2 ] ]

A morphism in the category of projective graded right modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
x_1,x_1,
x_2,x_2
(over a graded ring)

A projective graded right module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) of rank 2 and degrees:
[ [ 0, 2 ] ]
```

```
================================================================================
```

6.4 A few categorical constructions for the presentation category over the left modules

Example

```
gap> dSl := DirectSum( [ presentation1l, presentation2l ] );
<An object of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> IdentityMorphism( dSl );
<A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> ZeroMorphism( ZeroObject( CapCategory( presentation1l ) ), dSl );
<A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> p1l := ProjectionInFactorOfDirectSumWithGivenDirectSum(
>         [ presentation1l, presentation2l ], 1, dSl );
<A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( p1l );
A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
0,1,
0,0,
0,0
(over a graded ring)
gap> p2l := ProjectionInFactorOfDirectSumWithGivenDirectSum(
>         [ presentation1l, presentation2l ], 2, dSl );
<A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( p2l );
A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
0,0,
0,0,
1,0,
0,1
```

```
(over a graded ring)
gap> i1_l := InjectionOfCofactorOfDirectSumWithGivenDirectSum(
>        [ presentation1l, presentation2l ], 1, dSl );
<A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( i1_l );
A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,0,0,
0,1,0,0
(over a graded ring)
gap> i2_l := InjectionOfCofactorOfDirectSumWithGivenDirectSum(
>        [ presentation1l, presentation2l ], 2, dSl );
<A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( i2_l );
A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
0,0,1,0,
0,0,0,1
(over a graded ring)
gap> source_l := CAPCategoryOfProjectiveGradedLeftModulesObject(
>             [ [[1,0], 3 ] ], S );
<A projective graded left module of rank 3>
gap> Display( source_l );
A projective graded left module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
of rank 3 and degrees:
[ [ ( 1, 0 ), 3 ] ]
gap> range_l := CAPCategoryOfProjectiveGradedLeftModulesObject(
>            [ [[0,0], 2 ] ], S );
<A projective graded left module of rank 2>
gap> Display( range_l );
A projective graded left module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
of rank 2 and degrees:
[ [ 0, 2 ] ]
gap> test_morphism_l := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>     source_l,
>     HomalgMatrix( [[ "x_1", "x_2"], [ "x_1", "x_2"], [ "x_1", "x_2"]], S),
>     range_l );
<A morphism in the category of projective graded left modules over Q[x_1,x_2,x_3,x_4]
```

```
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [0, 1 ] ])>
gap> Display( test_morphism_l );
A morphism in the category of projective graded left modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [0, 1 ] ]) with matrix:
x_1,x_2,
x_1,x_2,
x_1,x_2
(over a graded ring)
gap> test_object_l := CAPPresentationCategoryObject( test_morphism_l );
<An object of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( test_object_l );
An object of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
x_1,x_2,
x_1,x_2,
x_1,x_2
(over a graded ring)
gap> mor1_l := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>           range_l, HomalgMatrix( [[1,0],[0,1]] , S ), Q3 );
<A morphism in the category of projective graded left modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( mor1_l );
A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
0,1
(over a graded ring)
gap> mor1_l_presentation := CAPPresentationCategoryMorphism(
>                         test_object_l, mor1_l, presentation1l );
<A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( mor1_l_presentation );
A morphism of the presentation category over the CAP category of projective graded
left modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]).
Presentation:
A morphism in the category of projective graded left modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
1,0,
0,1
(over a graded ring)
gap> mor2_l := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>           range_l, HomalgMatrix( [[1,0],[0,1]] , S ), Q2 );
<A morphism in the category of projective graded left modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ]])>
```

```
gap> Display( mor2_l );
A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
0,1
(over a graded ring)
gap> mor2_l_presentation := CAPPresentationCategoryMorphism(
>                         test_object_l, mor2_l, presentation2l );
<A morphism of the presentation category over the CAP category of projective graded left
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( mor2_l_presentation );
A morphism of the presentation category over the CAP category of projective graded
left modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]).
Presentation:
A morphism in the category of projective graded left modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
1,0,
0,1
(over a graded ring)
gap> sink_l := [ mor1_l_presentation, mor2_l_presentation ];
[ <A morphism of the presentation category over the CAP category of projective graded left
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>,
<A morphism of the presentation category over the CAP category of projective graded left
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])> ]
gap> diagram_l := [ Range( UnderlyingMorphism( presentation1l ) ),
>                   Range( UnderlyingMorphism( presentation2l ) ) ];
[ <A projective graded left module of rank 2>,
  <A projective graded left module of rank 2> ]
gap> uni_l := UniversalMorphismIntoDirectSumWithGivenDirectSum( diagram_l,
>                                                               sink_l,
>                                                               dSl );
<A morphism of the presentation category over the CAP category of projective graded left
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( uni_l );
A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded left modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
1,0,1,0,
0,1,0,1
(over a graded ring)
gap> mor21_l := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>            Q3, HomalgMatrix( [[1,0],[0,1]] , S ), range_l );
<A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( mor21_l );
A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
```

```
0,1
(over a graded ring)
gap> mor21_l_presentation := CAPPresentationCategoryMorphism( presentation1l,
>                                                             mor21_l,
>                                                             test_object_l );
<A morphism of the presentation category over the CAP category of projective graded left
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( mor21_l_presentation );
A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
0,1
(over a graded ring)
gap> mor22_l := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>            Q2, HomalgMatrix( [[1,0],[0,1]] , S ), range_l );
<A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( mor22_l );
A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
0,1
(over a graded ring)
gap> mor22_l_presentation := CAPPresentationCategoryMorphism(
>                          presentation2l, mor22_l, test_object_l );
<A morphism of the presentation category over the CAP category of projective graded left
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( mor22_l_presentation );
A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
0,1
(over a graded ring)
gap> sink2_l := [ mor21_l_presentation , mor22_l_presentation ];
[ <A morphism of the presentation category over the CAP category of projective graded left
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>,
<A morphism of the presentation category over the CAP category of projective graded left
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])> ]
gap> inj_l := UniversalMorphismFromDirectSumWithGivenDirectSum( diagram_l,
>                                                               sink2_l,
>                                                               dSl );
<A morphism of the presentation category over the CAP category of projective graded left
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
```

```
gap> Display( inj_1 );
A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded left modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
0,1,
1,0,
0,1
(over a graded ring)
gap> kernel_1 := KernelEmbedding( presentation_morphism_1 );
<A morphism of the presentation category over the CAP category of projective graded left
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( kernel_1 );
A morphism of the presentation category over the CAP category of projective graded left
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]).
Presentation:
A morphism in the category of projective graded left modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
x_1,x_2
(over a graded ring)
gap> cokernel_1 := CokernelProjection( presentation_morphism_1 );
<A morphism of the presentation category over the CAP category of projective graded left
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( cokernel_1 );
A morphism of the presentation category over the CAP category of projective
graded left modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded left modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
1,0,
0,1
(over a graded ring)
gap> IsMonomorphism( kernel_1 );
true
gap> IsEpimorphism( kernel_1 );
false
gap> IsIsomorphism( kernel_1 );
false
gap> IsMonomorphism( cokernel_1 );
false
gap> IsEpimorphism( cokernel_1 );
true
gap> IsIsomorphism( cokernel_1 );
false
gap> comparer_1 := CAPPresentationCategoryMorphism(
>                    Source( i1_1 ),
>                    CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>                            Range( UnderlyingMorphism( Source( i1_1 ) ) ),
>                            HomalgMatrix( [[ 0,1,0,0],[0,0,1,0]], S ),
```

```
>                           Range( UnderlyingMorphism( Range( i1_l ) ) )
>                     )
>                     , Range( i1_l )
>                );
<A morphism of the presentation category over the CAP category of projective graded left
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> IsEqualForMorphisms( i1_l, comparer_l );
false
```

6.5 A few categorical constructions for the presentation category over the right modules

Example

```
gap> dSr := DirectSum( [ presentation1r, presentation2r ] );
<An object of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> IdentityMorphism( dSr );
<A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> ZeroMorphism( ZeroObject( CapCategory( presentation1r ) ), dSr );
<A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> p1r := ProjectionInFactorOfDirectSumWithGivenDirectSum(
>         [ presentation1r, presentation2r ], 1, dSr );
<A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( p1r );
A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,0,0,
0,1,0,0
(over a graded ring)
gap> p2r := ProjectionInFactorOfDirectSumWithGivenDirectSum(
>         [ presentation1r, presentation2r ], 2, dSr );
<A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( p2r );
A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
```

```
with matrix:
0,0,1,0,
0,0,0,1
(over a graded ring)
gap> i1_r := InjectionOfCofactorOfDirectSumWithGivenDirectSum(
>        [ presentation1r, presentation2r ], 1, dSr );
<A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( i1_r );
A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
0,1,
0,0,
0,0
(over a graded ring)
gap> i2_r := InjectionOfCofactorOfDirectSumWithGivenDirectSum(
>        [ presentation1r, presentation2r ], 2, dSr );
<A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( i2_r );
A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
0,0,
0,0,
1,0,
0,1
(over a graded ring)
gap> source_r := CAPCategoryOfProjectiveGradedRightModulesObject(
>             [ [[1,0], 3 ] ], S );
<A projective graded right module of rank 3>
gap> Display( source_r );
A projective graded right module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
of rank 3 and degrees:
[ [ ( 1, 0 ), 3 ] ]
gap> range_r := CAPCategoryOfProjectiveGradedRightModulesObject(
>            [ [[0,0], 2 ] ], S );
<A projective graded right module of rank 2>
gap> Display( range_r );
A projective graded right module over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
```

```
of rank 2 and degrees:
[ [ 0, 2 ] ]
gap> test_morphism_r := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>     source_r,
>     HomalgMatrix( [[ "x_1", "x_1", "x_1" ], [ "x_2", "x_2", "x_2" ]], S),
>     range_r );
<A morphism in the category of projective graded right modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [0, 1 ] ])>
gap> Display( test_morphism_r );
A morphism in the category of projective graded right modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [0, 1 ] ]) with matrix:
x_1,x_1,x_1,
x_2,x_2,x_2
(over a graded ring)
gap> test_object_r := CAPPresentationCategoryObject( test_morphism_r );
<An object of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( test_object_r );
An object of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
x_1,x_1,x_1,
x_2,x_2,x_2
(over a graded ring)
gap> mor1_r := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>           range_r, HomalgMatrix( [[1,0],[0,1]] , S ), P3 );
<A morphism in the category of projective graded right modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( mor1_r );
A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
0,1
(over a graded ring)
gap> mor1_r_presentation := CAPPresentationCategoryMorphism(
>                         test_object_r, mor1_r, presentation1r );
<A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( mor1_r_presentation );
A morphism of the presentation category over the CAP category of projective graded
right modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]).
Presentation:
A morphism in the category of projective graded right modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
1,0,
0,1
```

```
(over a graded ring)
gap> mor2_r := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>           range_r, HomalgMatrix( [[1,0],[0,1]] , S ), P2 );
<A morphism in the category of projective graded right modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ]])>
gap> Display( mor2_r );
A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
0,1
(over a graded ring)
gap> mor2_r_presentation := CAPPresentationCategoryMorphism(
>                         test_object_r, mor2_r, presentation2r );
<A morphism of the presentation category over the CAP category of projective graded right
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( mor2_r_presentation );
A morphism of the presentation category over the CAP category of projective graded
right modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]).
Presentation:
A morphism in the category of projective graded right modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
1,0,
0,1
(over a graded ring)
gap> sink_r := [ mor1_r_presentation, mor2_r_presentation ];
[ <A morphism of the presentation category over the CAP category of projective graded right
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>,
<A morphism of the presentation category over the CAP category of projective graded right
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])> ]
gap> diagram_r := [ Range( UnderlyingMorphism( presentation1r ) ),
>                Range( UnderlyingMorphism( presentation2r ) ) ];
[ <A projective graded right module of rank 2>,
  <A projective graded right module of rank 2> ]
gap> uni_r := UniversalMorphismIntoDirectSumWithGivenDirectSum( diagram_r,
>                                                               sink_r,
>                                                               dSr );
<A morphism of the presentation category over the CAP category of projective graded right
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( uni_r );
A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded right modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
1,0,
0,1,
1,0,
0,1
(over a graded ring)
gap> mor21_r := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>           P3, HomalgMatrix( [[1,0],[0,1]] , S ), range_r );
```

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<A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( mor21_r );
A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
0,1
(over a graded ring)
gap> mor21_r_presentation := CAPPresentationCategoryMorphism( presentation1r,
>                                                             mor21_r,
>                                                             test_object_r );
<A morphism of the presentation category over the CAP category of projective graded right
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( mor21_r_presentation );
A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
0,1
(over a graded ring)
gap> mor22_r := CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>            P2, HomalgMatrix( [[1,0],[0,1]] , S ), range_r );
<A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( mor22_r );
A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
0,1
(over a graded ring)
gap> mor22_r_presentation := CAPPresentationCategoryMorphism(
>                          presentation2r, mor22_r, test_object_r );
<A morphism of the presentation category over the CAP category of projective graded right
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( mor22_r_presentation );
A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,
0,1
(over a graded ring)
gap> sink2_r := [ mor21_r_presentation , mor22_r_presentation ];
[ <A morphism of the presentation category over the CAP category of projective graded right
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>,
```

```
<A morphism of the presentation category over the CAP category of projective graded right
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])> ]
gap> inj_r := UniversalMorphismFromDirectSumWithGivenDirectSum( diagram_r,
>                                                               sink2_r,
>                                                               dSr );
<A morphism of the presentation category over the CAP category of projective graded right
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( inj_r );
A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded right modules over
Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])
with matrix:
1,0,1,0,
0,1,0,1
(over a graded ring)
gap> kernel_r := KernelEmbedding( presentation_morphism_r );
<A morphism of the presentation category over the CAP category of projective graded right
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( kernel_r );
A morphism of the presentation category over the CAP category of projective graded right
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]).
Presentation:
A morphism in the category of projective graded right modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
x_1,
x_2
(over a graded ring)
gap> cokernel_r := CokernelProjection( presentation_morphism_r );
<A morphism of the presentation category over the CAP category of projective graded right
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> Display( cokernel_r );
A morphism of the presentation category over the CAP category of projective
graded right modules over Q[x_1,x_2,x_3,x_4] (with weights
[ [ 1,0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]). Presentation:
A morphism in the category of projective graded right modules over Q[x_1,x_2,x_3,x_4]
(with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ]) with matrix:
1,0,
0,1
(over a graded ring)
gap> IsMonomorphism( kernel_r );
true
gap> IsEpimorphism( kernel_r );
false
gap> IsIsomorphism( kernel_r );
false
gap> IsMonomorphism( cokernel_r );
false
gap> IsEpimorphism( cokernel_r );
true
gap> IsIsomorphism( cokernel_r );
```

```
false
gap> comparer_r := CAPPresentationCategoryMorphism(
>                     Source( i1_r ),
>                     CAPCategoryOfProjectiveGradedLeftOrRightModulesMorphism(
>                             Range( UnderlyingMorphism( Source( i1_r ) ) ),
>                             HomalgMatrix( [[0,0],[1,0],[0,1],[0,0]], S ),
>                             Range( UnderlyingMorphism( Range( i1_r ) ) )
>                     )
>                     , Range( i1_r )
>                );
<A morphism of the presentation category over the CAP category of projective graded right
modules over Q[x_1,x_2,x_3,x_4] (with weights [ [ 1, 0 ], [ 1, 0 ], [ 0, 1 ], [ 0, 1 ] ])>
gap> IsEqualForMorphisms( i1_r, comparer_r );
false
```

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