Intersecting D6-Brane Models

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Section 1

Intersecting D6-brane setup

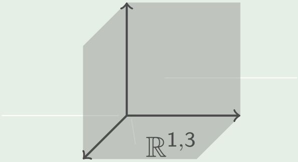
Internal and external space

Stategie

- $\mathbb{R}^{1,9} = \mathbb{R}^{1,3} \times \mathbb{R}^6$
- cover external space $\mathbb{R}^{1,3}$ by each D6-brane

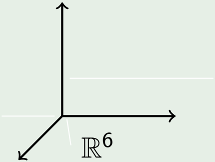
$\Rightarrow$ D6-branes 3-dimensional in internal space $\mathbb{R}^6$

Picture



(a) 4 dim. of branes

$\times$



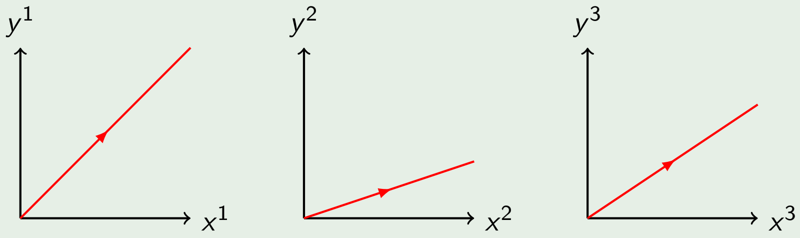
(b) 3 dim. of branes

Separation of the internal space

Factorizable branes

- $\mathbb{R}^6 = \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$
- Our choice - each D6-brane is a **line** in each $\mathbb{R}^2$

Picture



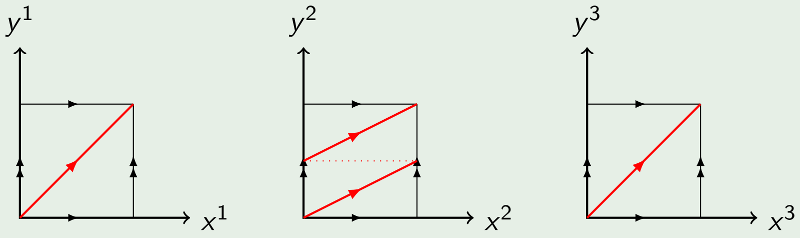
Toroidal compactification

Strategie

- Roll up each coordinate on circle

$\Rightarrow$ D6-brane becomes 3-cycle $\pi_a = \prod_{I=1}^{3} \left(n_a^I \left[a^I \right] + m_a^I \left[b^I \right] \right)$

Picture



Toroidal Compactification II

Topological intersection number

$$\pi_a \circ \pi_b = \prod_{I=1}^{3} \left(n_a^I m_b^I - n_b^I m_a^I \right)$$

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Example

- $\pi_a = (3,1) \times (1,0) \times (1,0)$
- $\pi_b = (0,1) \times (0,1) \times (0,1)$

$$\Rightarrow \pi_a \circ \pi_b = 3 \cdot 1 \cdot 1 = 3$$

Toroidal Compactification II

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Conclusion

- Multiple intersections possible

Stability conditions

Facts for D6-brane models

- (R-R tadpoles canceled) and (NS-NS tadpoles canceled)
  $\Leftrightarrow$ (R-R tadpoles canceled) and (model supersymmetric)

$\Rightarrow$ Requires orientifold

Stability conditions

Facts for D6-brane models

- (R-R tadpoles canceled) and (NS-NS tadpoles canceled) $\Leftrightarrow$ (R-R tadpoles canceled) and (model supersymmetric)

$\Rightarrow$ Requires orientifold

Orientifolding

- Define complex coordinate $z^I = x^I + iy^I$ on each T^2.
- Define involution $\overline{\sigma}$: $(z^1, z^2, z^3) \mapsto (\bar{z}^1, \bar{z}^2, \bar{z}^3)$
- Consider orientifold $\left(T^2 \times T^2 \times T^2\right) / (\overline{\sigma} \times \Omega)$

▸ More on the constraints

Section 2

Search for the Standard Model

Models on $T^2 \times T^2 \times T^2 / (\overline{\sigma} \times \Omega)$

Phenomenology
Wrapping numbers

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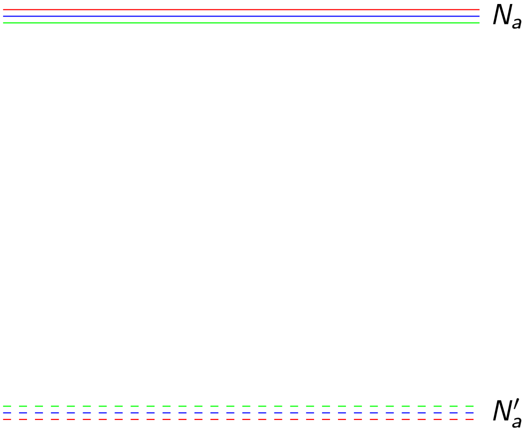
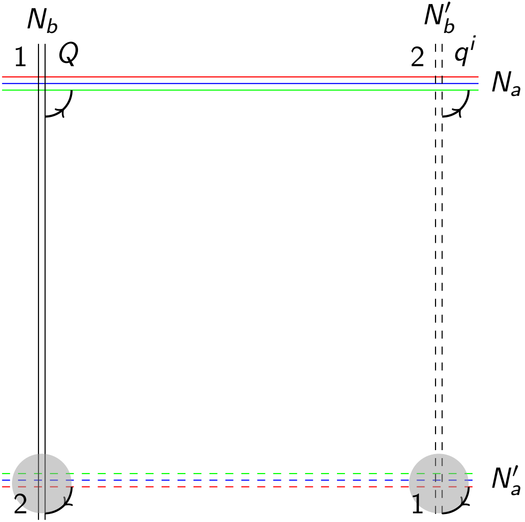
Phenomenology
Wrapping numbers



Models on $T^2 \times T^2 \times T^2/(\overline{\sigma} \times \Omega)$

Phenomenology Wrapping numbers



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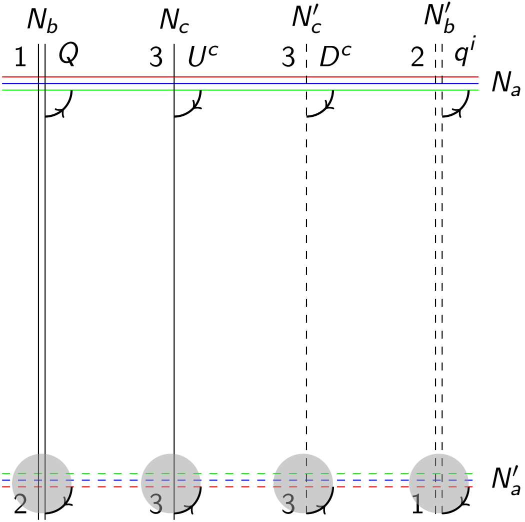
▸ Phenomenology ▸ Wrapping numbers



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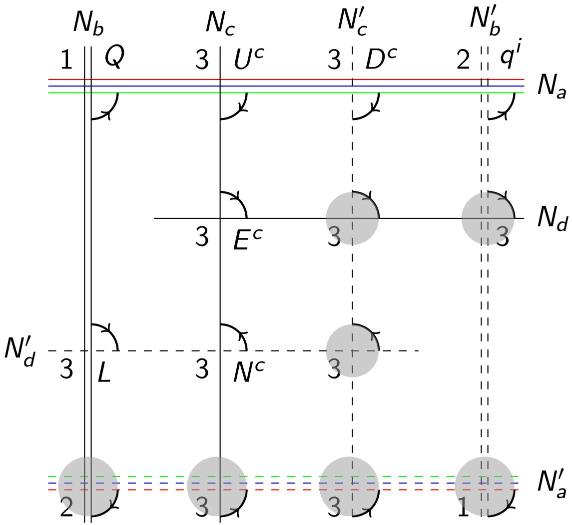

Models on $T^2 \times T^2 \times T^2/(\overline{\sigma} \times \Omega)$

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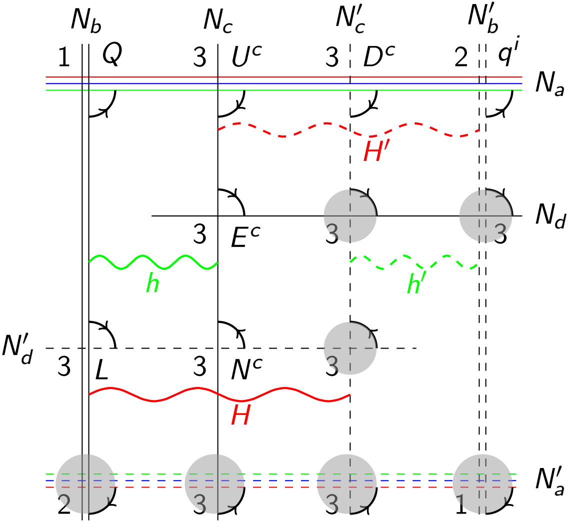

Models on different orientifolds

Example: $(T^2 \times T^2 \times T^2) / (\mathbb{Z}_2 \times \mathbb{Z}_2 \times \overline{\sigma} \times \Omega)$ ▸ More details

- 11 **semi-realistic** models constructed, meaning that e.g.
  - ✗ matter particles missing (or too many present)
  - ✗ exotic matter present

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Extension of search

- Different orientifolds
  - $(T^2 \times T^2 \times T^2) / (\mathbb{Z}_4 \times \overline{\sigma} \times \Omega)$
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Models on different orientifolds

Example: $\left(T^2 \times T^2 \times T^2\right) / (\mathbb{Z}_2 \times \mathbb{Z}_2 \times \overline{\sigma} \times \Omega)$ ▸ More details

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Extension of search

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  - $\left(T^2 \times T^2 \times T^2\right) / (\mathbb{Z}_2 \times \mathbb{Z}_4 \times \overline{\sigma} \times \Omega)$

$\Rightarrow$ Also semi-realistic models found

Conclusion on D6-brane models

Pros

- Standard Model like structures
- Unification with GR possible
- Prediction of gauge couplings

Conclusion on D6-brane models

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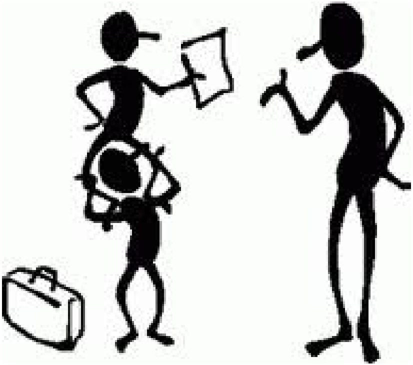
- Standard Model like structures
- Unification with GR possible
- Prediction of gauge couplings

Cons

- Only semi-realistic

Thank you for your attention!



Stability Conditions

Cancelation of R-R tadpoles

- $\sum_a N_a \left(\pi_a + \pi'_a\right) - 4\pi_{O6} = 0$

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Cancelation of R-R tadpoles

- $\sum_a N_a \left(\pi_a + \pi'_a\right) - 4\pi_{O6} = 0$
- **But** R-R charges classified by K-theory groups (rather than homology groups)

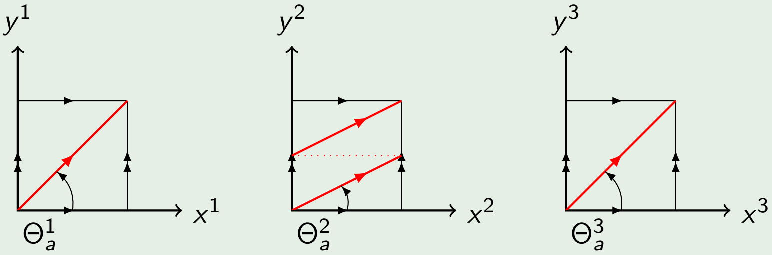
$\Rightarrow$ Require in addition even number of $USp\left(2, \mathbb{C}\right)$ fundamentals

Stability Conditions II

Supersymmetry condition

- Supersymmetry constraint: $\sum_{I=1}^{3} \Theta_a^I = 0 \mod 2\pi$

Picture



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ab-sector

Definition

- Strings from π_a to π_b form **ab-sector**

ab-sector

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Properties

- $U(N_a) - U(N_b)$ bifundamentals in ab-sector
- Ramond ground state is massless, chiral fermion
- Tension forces ab-sector strings to locate at intersection

$\Rightarrow$ Propatation **only** in the external space $\mathbb{R}^{1,3}$

- multiple intersection $\pi_a \circ \pi_b = 3$ is possible

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Conclusion

- ab-sector can give rise to **matter particles**

aa-sector

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- Strings from π_a to π_a form **aa-sector**

Properties

- Adjoint representations of $U(N_a)$
- Neveu-Schwarz ground state is massless boson
- Location not fixed in $T^2 \times T^2 \times T^2$

$\Rightarrow$ Winding and KK-states can appear

aa-sector

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Properties

- Adjoint representations of $U(N_a)$
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Conclusion

- aa-sector can give rise to Standard Model **gauge bosons**

Family replication in intersecting D6-brane models

Topological intersection number

- Define
$$\left[a^I\right] \circ \left[b^J\right] = -\left[b^J\right] \circ \left[a^I\right] = \delta^{IJ}$$
All other intersections vanish.
- Then for two 3-cycles
  - $\pi_a = \prod_{I=1}^{3} \left(n_a^I \left[a^I\right] + m_a^I \left[b^I\right]\right)$
  - $\pi_b = \prod_{I=1}^{3} \left(n_b^I \left[a^I\right] + m_b^I \left[b^I\right]\right)$

  the topological intersection number is
$$\pi_a \circ \pi_b = \prod_{I=1}^{3} \left(n_a^I m_b^I - n_b^I m_a^I\right)$$

Family replication in intersecting D6-brane models II

Example

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Family replication in intersecting D6-brane models II

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$$\Rightarrow \pi_a \circ \pi_b = 3 \cdot (-1) \cdot (-1) = 3$$

Consequence

- Multiple intersections possible

$\Rightarrow$ Integrates family replication into intersecting D6-brane models

Masses For Strings

General formula

$$\alpha' M^2 = N_{\perp,\nu} + \frac{Y^2}{4\pi^2\alpha'} + \nu \cdot \sum_{I=1}^{3} \left|\vartheta_{ab}^I\right| - \nu$$

- $Y \widehat{=}$ length of string
- $\nu = \begin{cases} 0 & \text{Ramond sector} \\ \frac{1}{2} & \text{Neveu-Schwarz sector} \end{cases}$
- $\vartheta_{ab}^I \widehat{=}$ intersection angle in I-th two-torus

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Example

Ground state in NS-sector has $2\alpha' M^2 = \sum_{I=1}^{3} \left|\vartheta^I_{ab}\right| - 1$

Yukawa couplings

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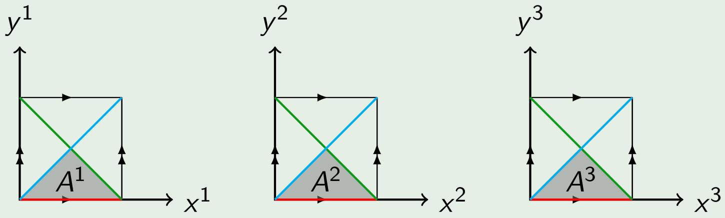
General Features

- 2 fermions and Higgs doublet located at different brane intersections

$\Rightarrow$ Triangular worldsheet governs interaction

$$Y \sim \exp\left(-A^1\right) \cdot \exp\left(-A^2\right) \cdot \exp\left(-A^3\right)$$

Picture



Models on $\left(T^2 \times T^2 \times T^2\right) / (\overline{\sigma} \times \Omega)$

Wrapping numbers

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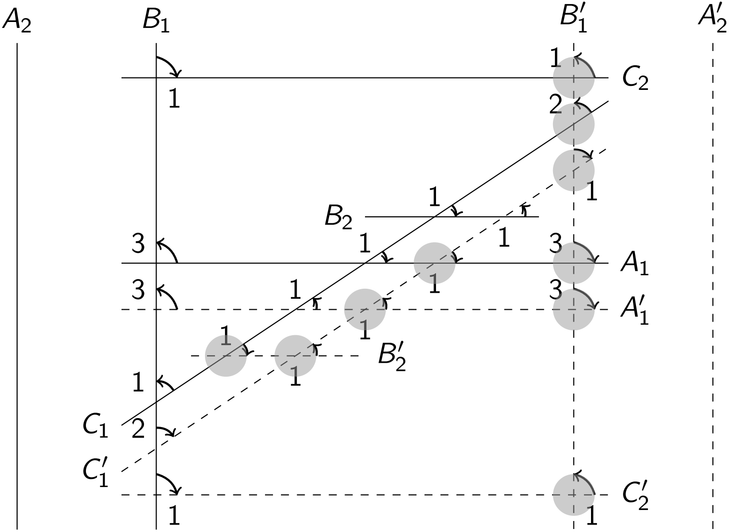
Brane	Wrapping Numbers	Gauge Group
$N_a = 3$	$\left(\frac{1}{\beta^1}, 0\right) \times \left(n_a^2, \epsilon\beta^2\right) \times \left(\frac{1}{\rho}, \frac{1}{2}\right)$	$U(3)$
$N_a' = 3$	$\left(\frac{1}{\beta^1}, 0\right) \times \left(n_a^2, -\epsilon\beta^2\right) \times \left(\frac{1}{\rho}, -\frac{1}{2}\right)$	
$N_b = 2$	$\left(n_b^1, -\epsilon\beta^1\right) \times \left(\frac{1}{\beta^2}, 0\right) \times \left(1, \frac{3\rho}{2}\right)$	$U(2)$
$N_b' = 2$	$\left(n_b^1, \epsilon\beta^1\right) \times \left(\frac{1}{\beta^2}, 0\right) \times \left(1, -\frac{3\rho}{2}\right)$	
$N_c = 1$	$\left(n_c^1, 3\rho\epsilon\beta^1\right) \times \left(\frac{1}{\beta^2}, 0\right) \times (0, 1)$	$U(1)$
$N_c' = 1$	$\left(n_c^1, -3\rho\epsilon\beta^1\right) \times \left(\frac{1}{\beta^2}, 0\right) \times (0, -1)$	
$N_d = 1$	$\left(\frac{1}{\beta^1}, 0\right) \times \left(n_d^2, -\frac{\beta^2\epsilon}{\rho}\right) \times \left(1, \frac{3\rho}{2}\right)$	$U(1)$
$N_d' = 1$	$\left(\frac{1}{\beta^1}, 0\right) \times \left(n_d^2, \frac{\beta^2\epsilon}{\rho}\right) \times \left(1, -\frac{3\rho}{2}\right)$	

Model on $\left(T^2 \times T^2 \times T^2\right) / (\mathbb{Z}_2 \times \mathbb{Z}_2 \times \overline{\sigma} \times \Omega)$



Model on $\left(T^2 \times T^2 \times T^2\right) / \left(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \overline{\sigma} \times \Omega\right)$ II

Wrapping numbers of branes

Brane	$(n_a^1, m_a^1) \times (n_a^2, m_a^2) \times (n_a^3, \widetilde{m}_a^3)$	Gauge Group
$A_1 = 4$	$(0,1) \times (0,-1) \times \left(2, \widetilde{0}\right)$	$U(1)^2$
$A_2 = 1$	$(1,0) \times (1,0) \times \left(2, \widetilde{0}\right)$	$USp(2,\mathbb{C})_A$
$B_1 = 2$	$(1,0) \times (1,-1) \times \left(1, \widetilde{\frac{3}{2}}\right)$	$SU(2) \times U(1)$
$B_2 = 1$	$(1,0) \times (0,1) \times \left(0, \widetilde{-1}\right)$	$USp(2,\mathbb{C})_B$
$C_1 = 3 + 1$	$(1,1) \times (1,0) \times \left(1, \widetilde{\frac{1}{2}}\right)$	$SU(3) \times U(1)^2$
$C_2 = 2$	$(0,1) \times (1,0) \times \left(0, \widetilde{-1}\right)$	$USp(4,\mathbb{C})$

Model on $\left(T^2 \times T^2 \times T^2\right) / (\mathbb{Z}_2 \times \mathbb{Z}_2 \times \overline{\sigma} \times \Omega)$ III

Wrapping numbers of image branes

Brane	$(n_a^1, m_a^1) \times (n_a^2, m_a^2) \times (n_a^3, \widetilde{m}_a^3)$	Gauge Group
$A_1' = 4$	$(0,-1) \times (0,1) \times \left(2, \widetilde{0}\right)$	$U(1)^2$
$A_2' = 1$	$(1,0) \times (1,0) \times \left(2, \widetilde{0}\right)$	$USp(2,\mathbb{C})_A$
$B_1' = 2$	$(1,0) \times (1,1) \times \left(1, \widetilde{-\frac{3}{2}}\right)$	$SU(2) \times U(1)$
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$C_1' = 3+1$	$(1,1) \times (1,0) \times \left(1, \widetilde{-\frac{1}{2}}\right)$	$SU(3) \times U(1)^2$
$C_2' = 2$	$(0,-1) \times (1,0) \times \left(0, \widetilde{1}\right)$	$USp(4,\mathbb{C})$

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Classification of D6-Branes I

Label	(P, Q, R, S)	$\left(n_a^{1,o}, n_a^{2,o}, n_a^{3,o}\right)$	$\left(m_a^{1,o}, m_a^{2,o}, m_a^{3,o}\right)$
A1	$(-,+,+,+)$	$(+,+,-)$	$(+,+,-)$
A2	$(+,-,+,+)$	$(+,+,+)$	$(+,-,-)$
A3	$(+,+,-,+)$	$(+,+,+)$	$(-,+,-)$
A4	$(+,+,+,-)$	$(+,+,+)$	$(-,-,+)$
B1	$(+,+,0,0)$	$(1,+,+)$	$(0,+,-)$
B2	$(+,0,+,0)$	$(+,1,+)$	$(+,0,-)$
B3	$(+,0,0,+)$	$(+,+,1)$	$(+,-,0)$
B4	$(0,+,+,0)$	$(+,+,0)$	$(-,-,1)$
B5	$(0,+,0,+)$	$(+,0,+)$	$(-,1,-)$
B6	$(0,0,+,+)$	$(0,+,+)$	$(1,-,-)$

Classification of D6-Branes II

Label	(P, Q, R, S)	$\left(n_a^{1,o}, n_a^{2,o}, n_a^{3,o}\right)$	$\left(m_a^{1,o}, m_a^{2,o}, m_a^{3,o}\right)$
C1	$(1, 0, 0, 0)$	$(1, 1, 1)$	$(0, 0, 0)$
C2	$(0, 1, 0, 0)$	$(1, 0, 0)$	$(0, 1, -1)$
C3	$(0, 0, 1, 0)$	$(0, 1, 0)$	$(1, 0, -1)$
C4	$(0, 0, 0, 1)$	$(0, 0, 1)$	$(1, -1, 0)$

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