# *CAP, machine learning and string theory*

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CAP days 2018

# Overview

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## Outline

- Brief introduction to string theory
- Search for our universe: How can CAP help?
- Exploring the landscape with CAP and machine learning

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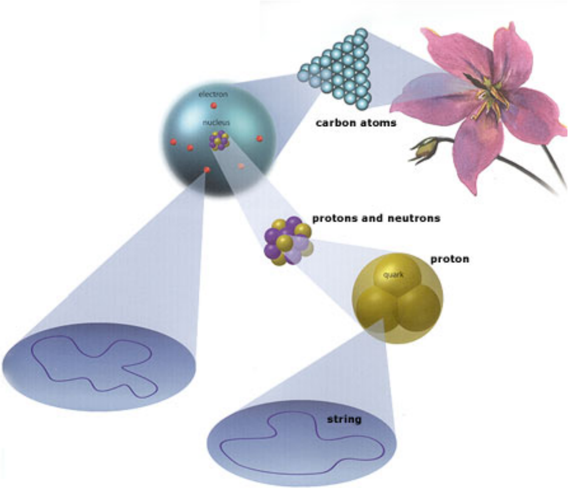
## Presentation based on work with ...

- T. Weigand and C. Mayrhofer 1706.04616, 1706.08528, 1802.08860
- M. Barakat, S. Gutsche, S. Posur, K. M. Saleh:
  5 CAP-packages on *https://github.com/HereAround*
- K. Veschgini in progress

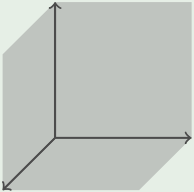
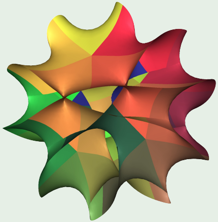
# Motivation: What if elementary particles were strings?




from 'A Layman's Guide To String Theory'

# Consequence: Universe is 10-dimensional!

## Cartoon

$\mathcal{S} \; = $  $\times$ 

our 4-dim. world $\mathcal{W}$ — 'small' 6-dim. manifold $\mathcal{M}_6$

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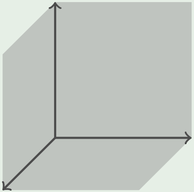
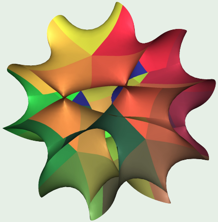
$\mathcal{S} \; = $  $\times$ 

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## Summary

- Universe: 10 dimensional manifold $\mathcal{S}$
- Compactification: $\mathcal{S} = \mathcal{W} \times \mathcal{M}_6$ and $\mathcal{M}_6$ compact

# Consequence II: String theory has many solutions!

## Ambiguity: Which manifold $\mathcal{M}_6$ to choose?

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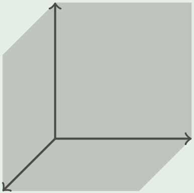
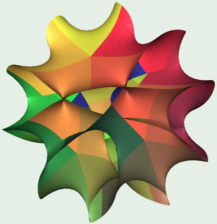


$\times$



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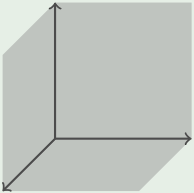

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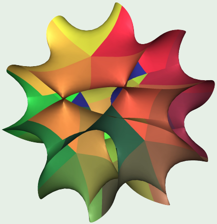


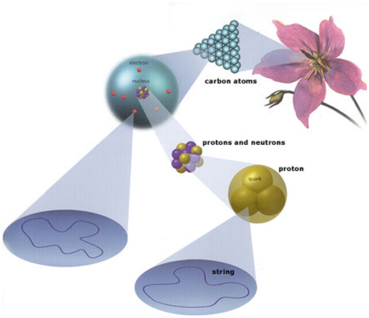
'small' 6-dim. manifold $\mathcal{M}_6$

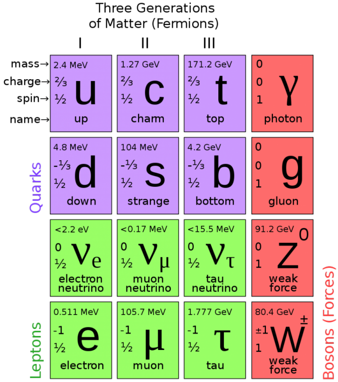
## Choices – curse or blessing?

- Many ($\sim 10^{1000}$) possible choices for $\mathcal{M}_6$
- Holy grail: Find $\mathcal{M}_6$ such that string theory on $\mathcal{S} = \mathcal{W} \times \mathcal{M}_6$ reproduces physics experienced in $\mathcal{W}$

# Status of search

# A quantity to count: Generations of fundamental particles



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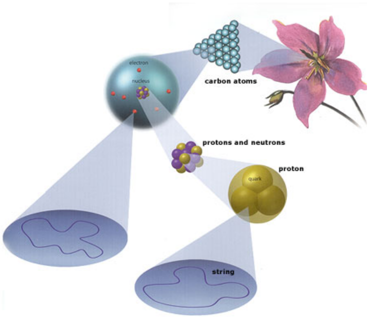
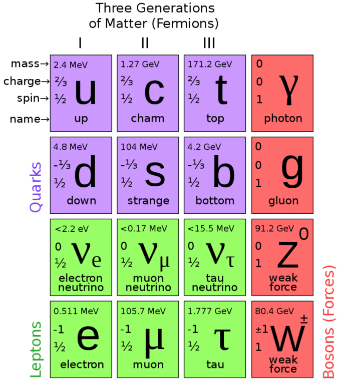


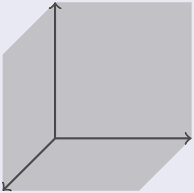
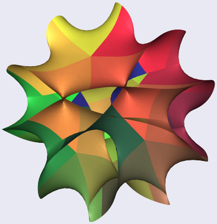




## Strategy

Count **number of generations** of (massless) particles!

# How to count generations of (massless) particles?

## Cartoon of compactification

$\mathcal{S} \; = $  $\times$ 

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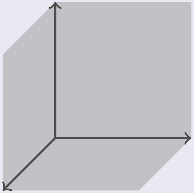
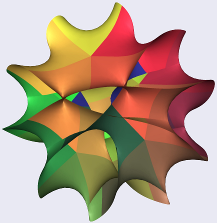
$\mathcal{S} =$  $\times$ 

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## Technicalities in a nutshell

- Quantum particles $\widehat{=}$ $\mathbb{C}$-valued functions on $\mathcal{S}$

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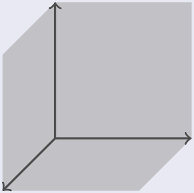

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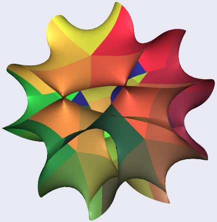

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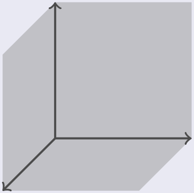
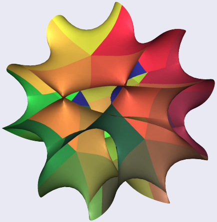
$\mathcal{S}$ $=$  $\times$ 

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## Technicalities in a nutshell

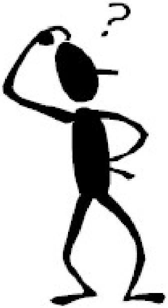
- Quantum particles $\widehat{=}$ $\mathbb{C}$-valued functions on $\mathcal{S}$

$\Rightarrow$ How many suitable $\mathbb{C}$-valued functions exist on $\mathcal{M}_6$?

$\Rightarrow$ Eventually: Compute sheaf cohomologies on $\mathcal{M}_6$

0403166, 0808.3621, 1106.4804, 1706.04616, 1802.08860 and many others

# Questions so far?



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1. Pick 'nice' class of manifolds $\mathcal{M}_6$
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4. Employ these categories to compute sheaf cohomologies

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## Revision: Defining data of toric varieties

- Cox ring $S = \mathbb{Q}\left[x_1, \ldots, x_n\right]$
- Homomorphism of monoids $\deg \colon \mathrm{Mons}\left(S\right) \to \mathbb{Z}^n$
- Stanley-Reissner ideal $I_{\mathrm{SR}} \subseteq S$

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## Example: $\mathbb{P}^2_{\mathbb{Q}}$

- $S = \mathbb{Q}[x_1, x_2, x_3]$
- $\deg \colon S \to \mathbb{Z}$ with $\deg(x_1) = \deg(x_2) = \deg(x_3) = 1$
- $I_{\mathrm{SR}} = \langle x_1 \cdot x_2 \cdot x_3 \rangle$

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## Sheafification functor

- $S\text{-fpgrmod}$: **category** of finitely presented graded $S$-modules
- $\mathfrak{Coh} X_\Sigma$: **category** of coherent sheaves on $X_\Sigma$

$\Rightarrow$ There exists the sheafification functor

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## Computer models for coherent sheaves

- The category $S$-fpgrmod can be handled with CAP

$\Rightarrow$ $S$-fpgrmod can serve as computer models for coherent sheaves

1003.1943, 1202.3337, 1210.1425, 1212.4068, 1409.2028, 1409.6100

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## Computer models for coherent sheaves

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⇒ $S\text{-fpgrmod}$ can serve as computer models for coherent sheaves

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## Strategy of implementation

1. Implement category of projective graded $S$-modules
2. 'Derive' $S\text{-fpgrmod}$ as Freyd category 1712.03492 and references therein

# $S$-fpgrmod 1 – Category of projective graded $S$-modules

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## Objects: $M = \bigoplus_{d \in I} S\left(d\right)$

- $I \subseteq \mathbb{Z}^n$ an indexing set
- *graded*, i. e. $S_i M_j \subseteq M_{i+j}$

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## Example: $S$ the Cox ring of $\mathbb{P}^2_{\mathbb{Q}}$

$\varphi \colon S(-1) \xrightarrow{(x_1)} S(0)$ is morphism in this category since

$$\underbrace{S(-1) \ni 1}_{\text{degree1}} \mapsto \varphi(1) = \underbrace{x_1 \in S(0)}_{\text{degree1}}$$

# $S$-fpgrmod 2: Objects

**General rule:**

Objects in $S\text{-fpgrmod} \mathrel{\widehat{=}}$ morphisms of projective graded $S$-modules

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**Example on $\mathbb{P}^2_{\mathbb{Q}}$: $S = \mathbb{Q}[x_1, x_2, x_3]$, $\deg(x_i) = 1$**

$M_\varphi \equiv \mathrm{coker}(\varphi)$ and $M_\psi \equiv \mathrm{coker}(\psi)$ are abstractly described by

$$\psi \colon S(-2)^{\oplus 3} \xrightarrow{R} S(-1)^{\oplus 3}, \quad R = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}, \quad \varphi \colon 0 \to S(0)$$

# $S$-fpgrmod 2: Objects

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## Notation

$$\begin{array}{c} S(-2)^{\oplus 3} \\ R \big\downarrow \\ S(-1)^{\oplus 3} \end{array} \qquad\qquad \begin{array}{c} 0 \\ \big\downarrow 0 \\ S(0) \end{array}$$

# $S$-fpgrmod 3: Morphisms

## Definition: Morphism $M_\psi \to M_\varphi$ is commutative diagram

$$\begin{array}{ccc} S(-2)^{\oplus 3} & \xrightarrow{A} & 0 \\ \downarrow R & & \downarrow 0 \\ S(-1)^{\oplus 3} & \xrightarrow{B} & S(0) \end{array}$$

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# $S$-fpgrmod 3: Morphisms

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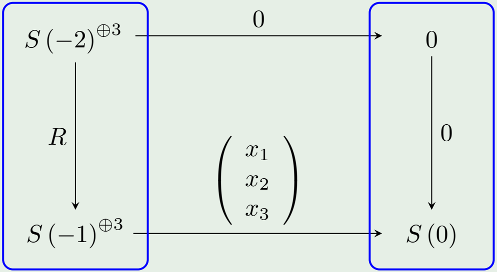


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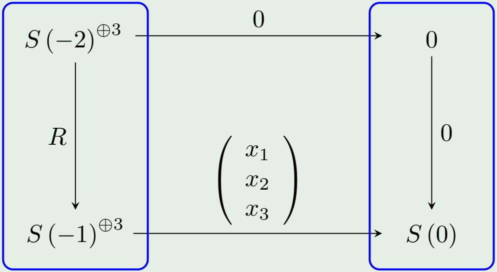


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## Implementation for CAP at *https://github.com/HereAround:*

- ‘CAPCategoryOfProjectiveGradedModules’
- ‘CAPPresentationCategory’
- ‘PresentationByProjectiveGradedModules’

# Computing $H^0$ – general idea

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**Definition**

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## Definition

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## Idea

- $M$ such that $\widetilde{M} \cong \mathcal{O}_X$
- $F$ such that $\widetilde{F} \cong \mathcal{F}$

$\Rightarrow \Gamma\left(\mathscr{H}om_{\mathcal{O}_X}(\mathcal{O}_X, \mathcal{F})\right) \stackrel{?}{=} \mathrm{Hom}_S(M, F)_0$

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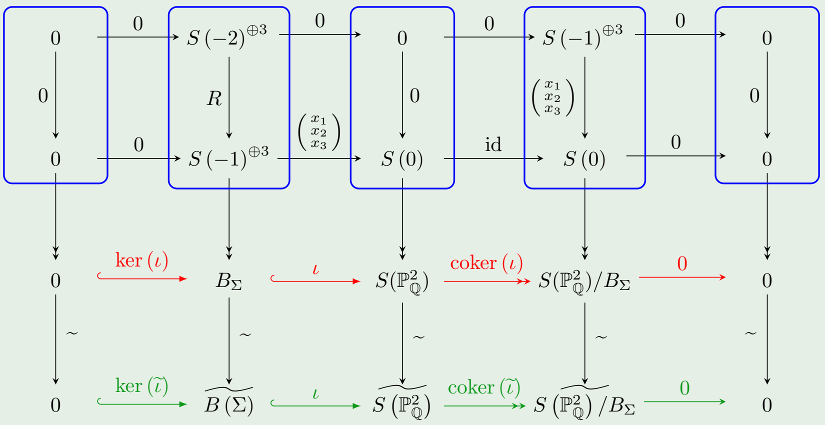
$\Rightarrow \Gamma(\mathscr{H}om_{\mathcal{O}_X}(\mathcal{O}_X, \mathcal{F})) \stackrel{?}{=} \mathrm{Hom}_S(M, F)_0$

## Careful!

In general wrong – have to choose $M$ carefully

# Computing $H^0$ – different models for the structure sheaf

**Example:** $B_\Sigma = \langle x_1, x_2, x_3 \rangle$ and $S$ are models for $\mathcal{O}_{\mathbb{P}^2_{\mathbb{Q}}}$



# Computing $H^0$ – is $B_\Sigma$ or $S$ better?

## Task

- On $\mathbb{P}^2_{\mathbb{Q}}$, $F = B_\Sigma = \langle x_1, x_2, x_3 \rangle$ satisfies $\widetilde{F} \cong \mathcal{O}_{\mathbb{P}^2_{\mathbb{Q}}}$

$\Rightarrow$ $H^0(\mathbb{P}^2_{\mathbb{Q}}, \widetilde{F}) \cong \mathbb{Q}^1$

$\Rightarrow$ Task: Reproduce this from $\mathrm{Hom}_S(X, F)_0$ with $X \in \{S, B_\Sigma\}$

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## Try 1: $X = S$

$\mathrm{Hom}_S(S, F)_0 \cong \mathbb{Q}^0$ – wrong result!

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## Try 2: $X = B_\Sigma$

$\mathrm{Hom}_S(B_\Sigma, F)_0 \cong \mathbb{Q}^1$ – correct result!

# Implemented Algorithm

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## Input and Output

- smooth, complete toric variety $X_\Sigma$
- $F \in S\text{-fpgrmod}$

$$\Longrightarrow \quad h^i\left(X_\Sigma, \widetilde{F}\right)$$

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- smooth, complete toric variety $X_\Sigma$
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$\Longrightarrow$ $h^i\left(X_\Sigma, \widetilde{F}\right)$

## Step-by-step (References in two slides)

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$$V^k(X_\Sigma) := \left\{L \in \mathrm{Pic}(X_\Sigma)\ ,\ h^k(X_\Sigma, L) = 0\right\}$$

# Implemented Algorithm

## Input and Output

- smooth, complete toric variety $X_\Sigma$
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# $SU(5) \times U(1)$-Tate model from 1706.04616

## Input and Output

- $C_{\mathbf{5}_{-2}} \subseteq \mathbb{P}^2_{\mathbb{Q}}$
- $L_{\mathbf{5}_{-2}} \leftrightarrow F$ and $F$ defined by $S(-36) \oplus S(-39) \oplus S(-41) \oplus S(-23) \oplus S(-38) \to S(-6) \oplus S(-21) \twoheadrightarrow F \to 0$

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## Apply Algorithm

1. Compute vanishing sets via *cohomCalg*:
$V^0(\mathbb{P}^2_{\mathbb{Q}}) = (-\infty, -1]_{\mathbb{Z}}$, $V^1(\mathbb{P}^2_{\mathbb{Q}}) = \mathbb{Z}$, $V^2(\mathbb{P}^2_{\mathbb{Q}}) = [-2, \infty)_{\mathbb{Z}}$

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2. Use vanishing sets to find ideal $I$ (along idea of G. Smith): $I = B_{\Sigma}^{(44)} \equiv \langle x_0^{44}, x_1^{44}, x_2^{44} \rangle$

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$$\mathrm{Ext}^1_S\left(B_\Sigma^{(44)}, F\right)_0$$

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3. $\mathbb{Q}^{37425} \rightarrow \mathbb{Q}^{27201} \twoheadrightarrow \mathrm{Ext}^1_S\left(B_\Sigma^{(44)}, F\right)_0 \rightarrow 0$

$$\Rightarrow 28 = \dim_{\mathbb{Q}}\left[\mathrm{Ext}^1_S\left(B_\Sigma^{(44)}, F\right)_0\right] = h^1\left(\mathbb{P}^2_{\mathbb{Q}}, \widetilde{F}\right)$$

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- Have combined
  - *cohomCalg* by R. Blumenhagen et al.
    1003.5217, 1006.0780, 1006.2392, 1010.3717
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$\Rightarrow$ Modern algorithm to compute sheaf cohomologies of **all** coherent sheaves on smooth, complete toric varieties

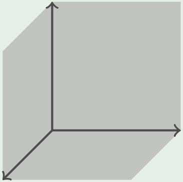
$\Rightarrow$ Many applications in string theory

# Questions so far?

# Moduli dependence of sheaf cohomologies

## Recall: String landscape = manifold $\mathcal{M}_6$ and substructure

$\mathcal{S}$ =



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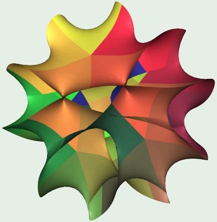

our 4-dim. world $\mathcal{W}$ ‘small’ 6-dim. manifold $\mathcal{M}_6$

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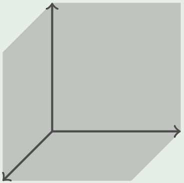
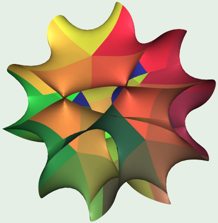
$\mathcal{S}$ =  $\times$ 

our 4-dim. world $\mathcal{W}$ — 'small' 6-dim. manifold $\mathcal{M}_6$

**Strategies**

- So far: **One choice** of manifold $\mathcal{M}_6$ **with substructure**

$\Rightarrow$ CAP allows to count number of generations $n_g$

- Now: How does $n_g$ vary if we alter $\mathcal{M}_6$ and its substructure?

# Moduli dependence of sheaf cohomologies II

## Example on substructure: $SU(5) \times U(1)_Y$-Tate model

- Substruc. $\supset C_{\mathbf{5}_{-2}} = V(\widetilde{a_{1,0}} \cdot \widetilde{a_{4,3}} - \widetilde{a_{3,2}} \cdot \widetilde{a_{2,1}}) \subseteq \mathbb{P}^2_{\mathbb{Q}}$
- $\widetilde{a_{1,0}} = c_1 x_1^4 + c_2 x_1^3 x_2 + c_3 x_1^2 x_2 x_3 + \ldots \in \mathbb{Q}[x_1, x_2, x_3]$
- $\deg \widetilde{a_{1,0}} = 7$, $\deg \widetilde{a_{2,1}} = 7$, $\deg \widetilde{a_{3,2}} = 10$, $\deg \widetilde{a_{4,3}} = 13$

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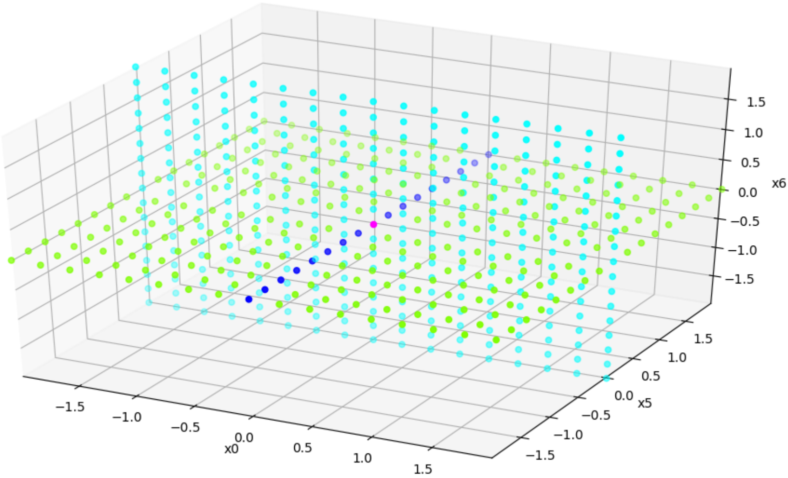
## $SU(5)xU(1)$-Tate Model from 1706.04616 ($\mathbf{R} = \mathbf{5}_{-2}$)

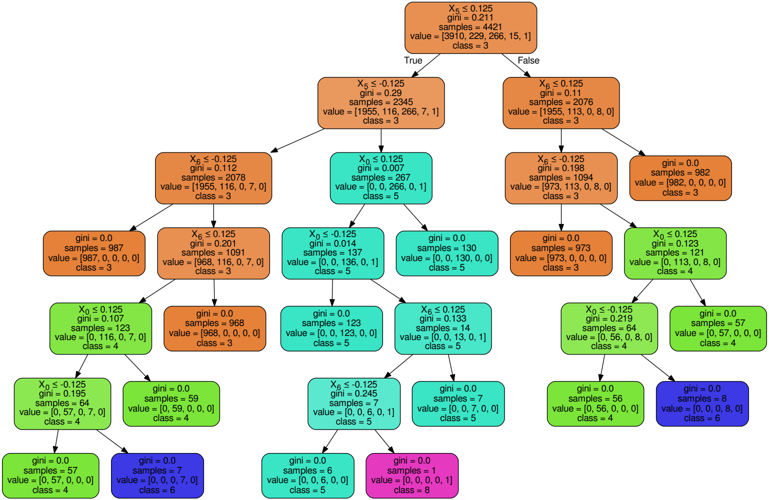
| | $\widetilde{a_{1,0}}$ | $\widetilde{a_{2,1}}$ | $\widetilde{a_{3,2}}$ | $\widetilde{a_{4,3}}$ | $h^0(C_{\mathbf{R}}, L_{\mathbf{R}})$ |
|---|---|---|---|---|---|
| $M_1$ | $(x_1 - x_2)^4$ | $x_1^7$ | $x_2^{10}$ | $x_3^{13}$ | 22 |
| $M_2$ | $(x_1 - x_2)\, x_3^3$ | $x_1^7$ | $x_2^{10}$ | $x_3^{13}$ | 21 |
| $M_3$ | $x_3^4$ | $x_1^7$ | $x_2^7 (x_1 + x_2)^3$ | $x_3^{12} (x_1 - x_2)$ | 11 |
| $M_4$ | $(x_1 - x_2)^3 x_3$ | $x_1^7$ | $x_2^{10}$ | $x_3^{13}$ | 9 |
| $M_5$ | $x_3^4$ | $x_1^7$ | $x_2^8 (x_1 + x_2)^2$ | $x_3^{11} (x_1 - x_2)^2$ | 7 |
| $M_6$ | $x_3^4$ | $x_1^7$ | $x_2^{10}$ | $x_3^8 (x_1 - x_2)^5$ | 6 |
| $M_7$ | $x_3^4$ | $x_1^7$ | $x_2^9 (x_1 + x_2)$ | $x_3^{10} (x_1 - x_2)^3$ | 5 |

# Other example: $dP_3$-example from 1802.08860




4913 data points, 3193 used for training, 1720 correctly predicted

# Other example: $dP_3$-example from 1802.08860 II



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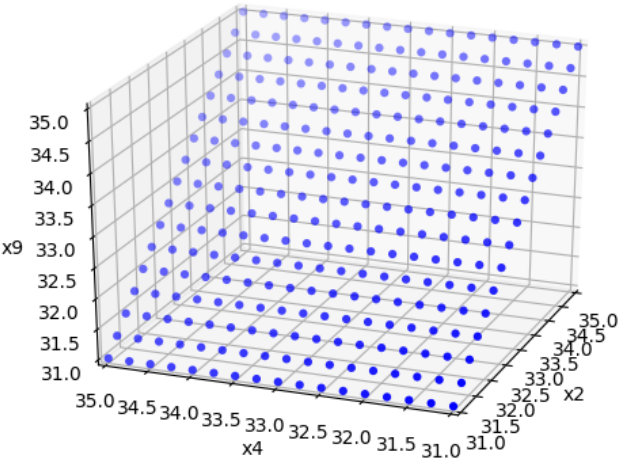
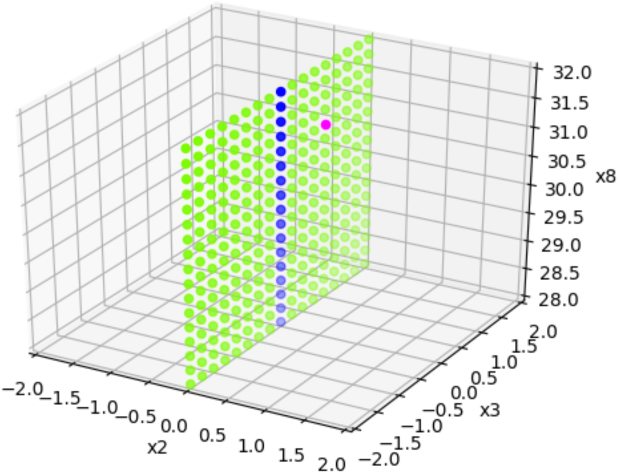


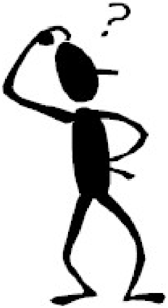

4913 data points, 4910 used for training, 3 correctly predicted

# Other example: $dP_3$-example from 1802.08860 IV




4913 data points, 2947 used for training, 1966 correctly predicted

# Questions so far?



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- Sheaf cohomology feature prominently in string theory

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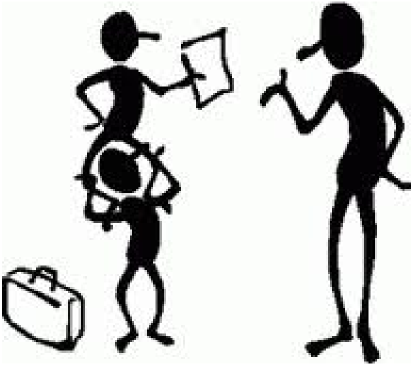
- Implementations available at *https://github.com/HereAround*
- Current effort: Open door for statistical analysis on the string landscape via machine learning

$\Rightarrow$ Decision trees seem to help us with this task

# Thank you for your attention!



# Schematic Picture: Physics and Geometry of F-theory

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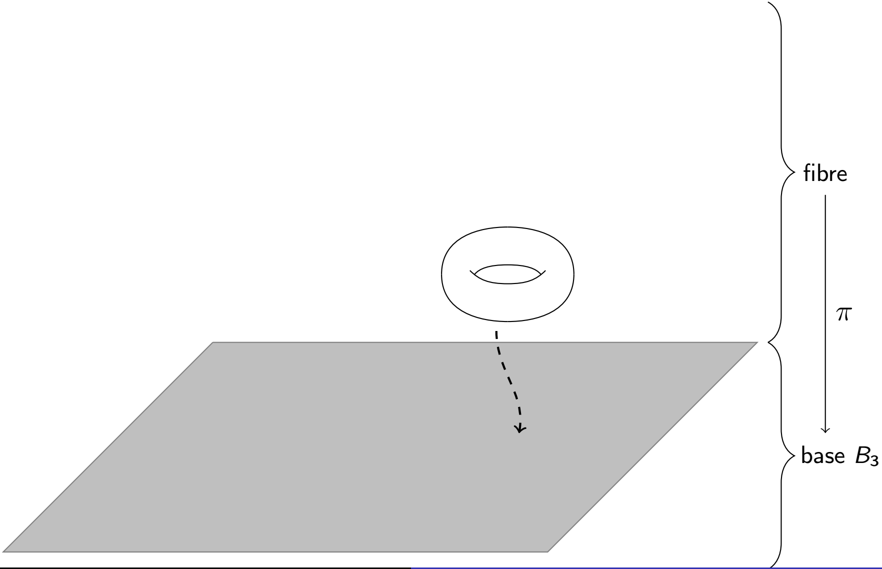

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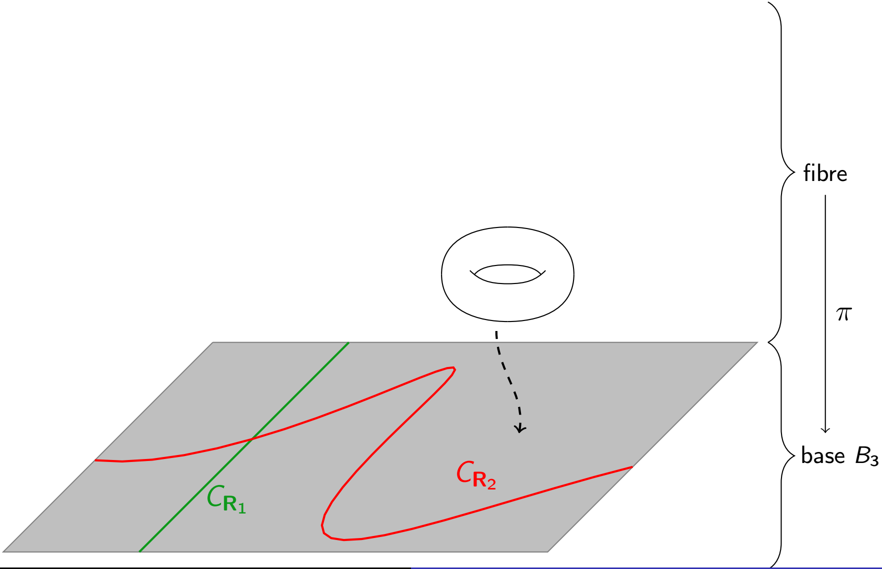

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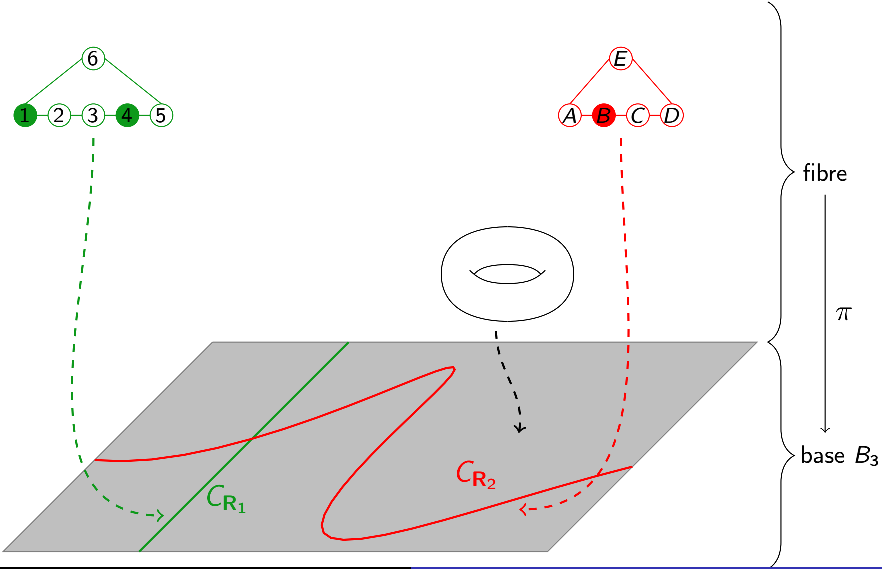

# From Divisors to Modules

## Input and Output

- $C = V(g_1, \ldots, g_k) \subseteq X_\Sigma$
- $D = V(f_1, \ldots, f_n) \in \mathrm{Div}(C)$

$\Longrightarrow$

$M$ s.t. $\mathrm{supp}(\widetilde{M}) = C$ and $\widetilde{M}|_C \cong \mathcal{O}_C(-D)$

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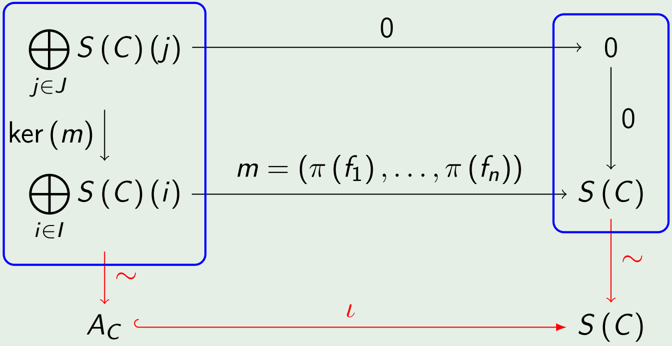
## Step 1: $S(C) := S/\langle g_1, \ldots, g_k \rangle$, $\pi \colon S \twoheadrightarrow S(C)$



# From Divisors to Modules II

## Step 2: Extend by zero to coherent sheaf on $X_\Sigma$

$$\left( \begin{array}{c} \bigoplus_{j \in J} S(j) \\ \Big\downarrow {\scriptstyle \ker(m)'} \\ \bigoplus_{i \in I} S(i) \end{array} \right) \Big\downarrow \sim \; A \qquad \otimes \qquad \left( \begin{array}{c} \bigoplus_{k \in K} S(k) \\ \Big\downarrow {\scriptstyle \begin{pmatrix} g_1 \\ \vdots \\ g_k \end{pmatrix}} \\ S(C) \end{array} \right) \Big\downarrow \sim \; B$$

$\Rightarrow M = A \otimes B$ satisfies $\mathrm{Supp}(\widetilde{M}) = C$ and $\widetilde{M}|_C \cong \mathcal{O}_C(-D)$

# From Divisors to Modules III

## Input and Output

- $C = V(g_1, \ldots, g_k) \subseteq X_\Sigma$
- $D = V(f_1, \ldots, f_n) \in \mathrm{Div}(C)$

$\Longrightarrow$

$M$ s.t. $\mathrm{supp}(\widetilde{M}) = C$ and $\widetilde{M}|_C \cong \mathcal{O}_C(+D)$

## Strategy

1. Compute $A_C$
2. Dualise via $A_C^\vee := \mathrm{Hom}_{S(C)}\left(S\left(C\right), A_C\right)$
3. Extend by zero by considering $A^\vee \otimes B$

$\Rightarrow$ $M^\vee := A^\vee \otimes B$ satisfies $\mathrm{Supp}(\widetilde{M}) = C$ and $\widetilde{M}|_C \cong \mathcal{O}_C\left(+D\right)$

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## Affine open cover

- Toric variety $X_\Sigma$ with Cox ring $S$

$\Rightarrow$ Covered by affine opens $\left\{ U_\sigma = \mathrm{Specm}\left(S_{(x^{\hat{\sigma}})}\right) \right\}_{\sigma \in \Sigma}$

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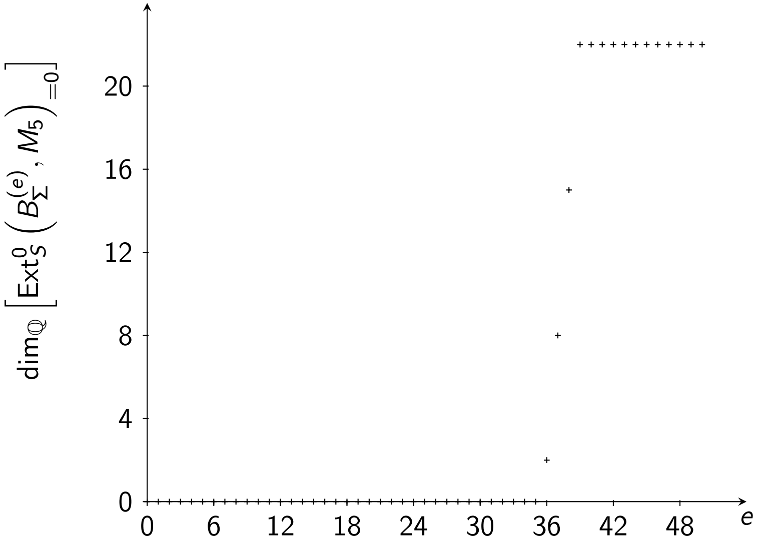
## Consequence

- $M_{(x^{\hat{\sigma}})} \leftrightarrow$ coherent sheaf on $U_\sigma = \text{Specm}(S_{(x^{\hat{\sigma}})})$
- local sections: $\widetilde{M_{(x^{\hat{\sigma}})}}(D(f)) = M_{(x^{\hat{\sigma}})} \otimes_{S_{(x^{\hat{\sigma}})}} \left( S_{(x^{\hat{\sigma}})} \right)_f$

# Module $M_5$ from 1706.04616: Quality Check I



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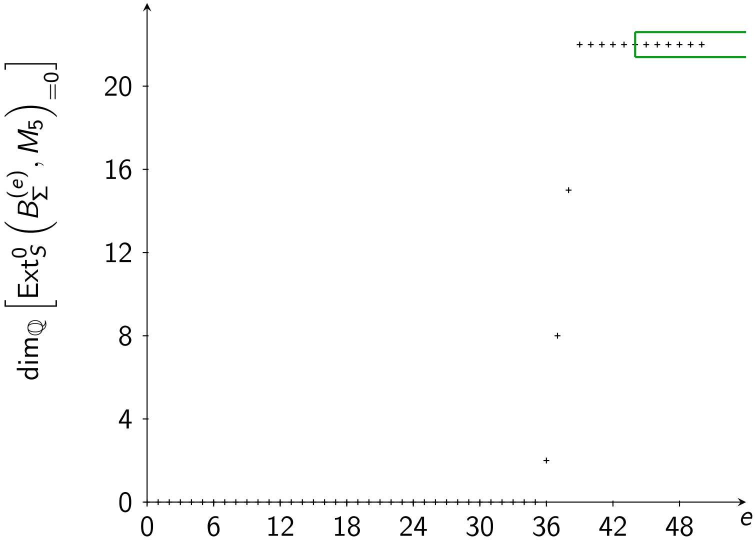

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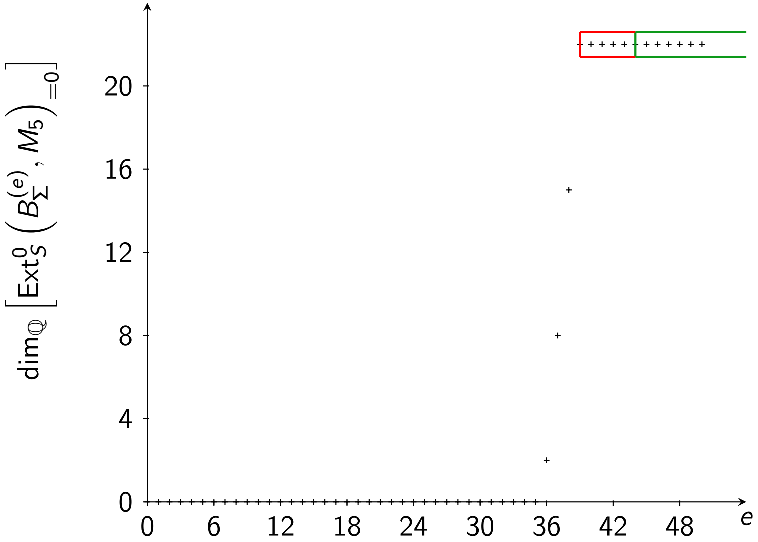
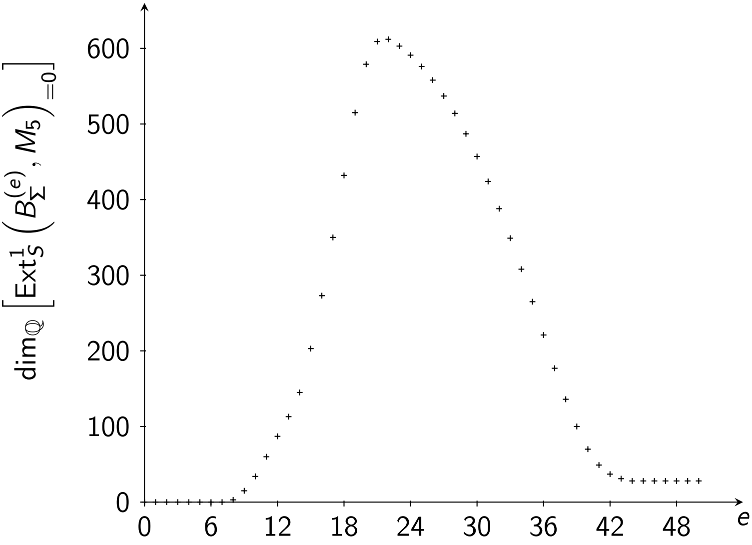


# Module $M_5$ from 1706.04616: Quality Check II



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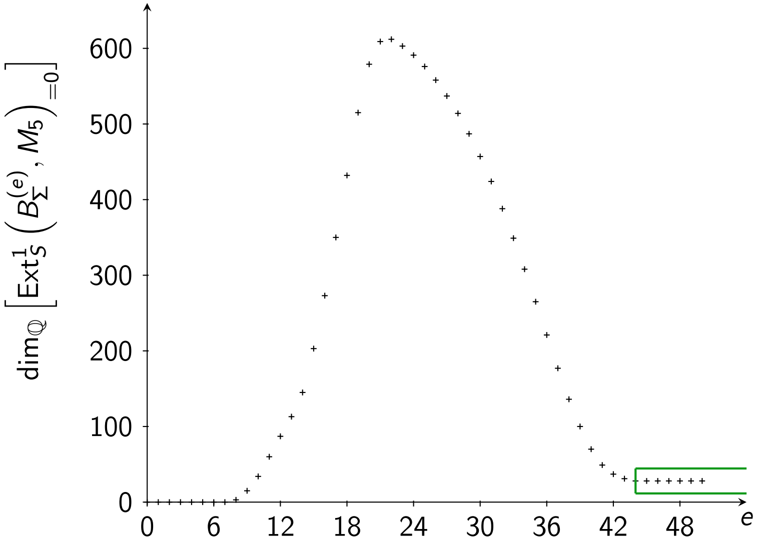

# How to determine the ideal $I$ in step 2 of algorithm?

**Input**

- $M \in S\text{-}\mathrm{fpgrmod}$
- $V^k(X_\Sigma) = \left\{ L \in \mathrm{Pic}(X_\Sigma)\ ,\ h^k(X_\Sigma, L) = 0 \right\}$

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## Preparation

- $p \in \mathrm{Cl}(X_\Sigma)$ **ample**, $m(p) = \{m_1, \ldots, m_k\}$ all monomials of degree $p$ and $I(p, e) = \left\langle m_1^e, \ldots, m_k^e \right\rangle$
- Pick $e = 0$ and increase it until subsequent conditions are met

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## How to find ideal $I$?

- Look at spectral sequence $\mathbb{E}_2^{p,q} \Rightarrow \mathrm{Ext}^{p+q}_{\mathcal{O}_{X_\Sigma}}\left( \widetilde{I(p,e)}, \widetilde{M} \right)$

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$\Rightarrow$ If no – increase $e$ until this is the case!

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$\Rightarrow$ Increase $e$ further until $H^m\left(\mathbf{C}^0\right) \cong \mathrm{Ext}^m_S(I(p,e), M)_0$

# The Hom-Embedding