Towards F-theory MSSMs

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With M. Cvetič, R. Donagi, M. Liu, M. Ong – 2102.10115, 2104.08297, 2205.00008

Motivation, goal, challenge and tool

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- Go beyond chiral spectrum of SM constructions. [Talk by W. Taylor and K. Li]
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  $\Rightarrow$ **Quadrillion F-theory standard models (QSMs)**. [Cvetič Halverson Lin Liu Tian '19]

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Compute vector-like spectra in reps. $(\overline{\mathbf{3}}, \mathbf{2})_{1/6}$, $(\overline{\mathbf{3}}, \mathbf{1})_{-2/3}$, $(\mathbf{1}, \mathbf{1})_1$ of F-theory QSMs.

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In global F-theory compactifications, vector-like spectra are **non-topological**.

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Tool

Root bundles (genearlizations of spin bundles) on **nodal curves**.

Chiral and desired vector-like spectra in the QSMs

Matter curve $C_{\mathbf{R}}$	$n_{\mathbf{R}} = \#$ chiral fields in rep $\mathbf{R}$	$\# \ n_{\overline{\mathbf{R}}} =$ chiral fields in rep $\overline{\mathbf{R}}$	Chiral index $\chi = n_{\mathbf{R}} - n_{\overline{\mathbf{R}}}$
$C_{(\mathbf{3},\mathbf{2})_{1/6}} = V(s_3, s_9)$			
$C_{(\mathbf{1},\mathbf{2})_{-1/2}} = V\left(s_3, s_2 s_5^2 + s_1(s_1 s_9 - s_5 s_6)\right)$			
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Matter curve $C_{\mathbf{R}}$	Necessary root bundle condition for $\mathcal{L}_{\mathbf{R}}$
$C_{(\mathbf{3},\mathbf{2})_{1/6}} = V(s_3, s_9)$	$\mathcal{L}_{(\mathbf{3},\mathbf{2})_{1/6}}^{\otimes 36} = K_{C_{(\mathbf{3},\mathbf{2})_{1/6}}}^{\otimes 24}$
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Exponents of root bundle constraints for base 3-folds B_3 with $K_{B_3}^3 = 18$. See [M.B. Cvetič Donagi Liu Ong '21] for exponents of B_3 with other $K_{B_3}^3$.

Necessary condition for $\mathcal{L}$: Root bundle constraints [M.B. Cvetič Donagi Liu Ong '21]

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- Constraints highly non-trivial:
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$\Rightarrow$ Agenda: Vector-like spectra of the QSMs from studying root bundles.

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  - There are exactly r^{2g} line bundles $\mathcal{L} \in \mathrm{Pic}(C)$ with $\mathcal{L}^r = T$.
  - Theory: Obtain all roots by twist one such $\mathcal{L}$ with r-torsion points of $\mathrm{Jac}(C)$.
  - **Practice: Tough.** Related: Discrete logarithm in Picard group of elliptic curve used for elliptic-curve cryptography).

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- **Nodal curve** $C^\bullet$ of genus g: [Jarves '98], [Caporaso Casagrande Cornalba '04]
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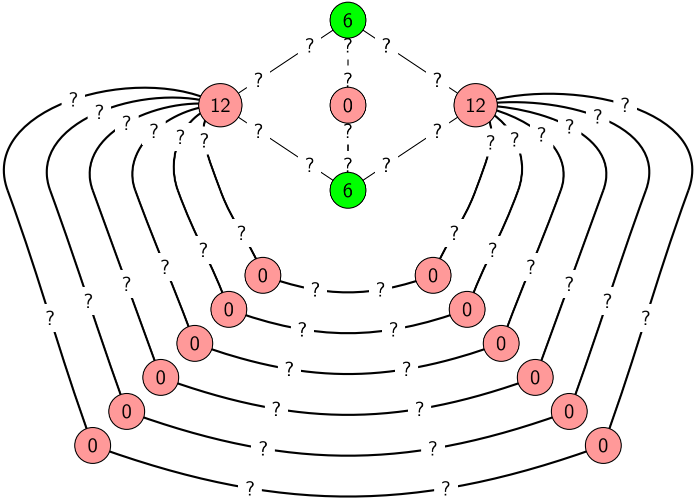
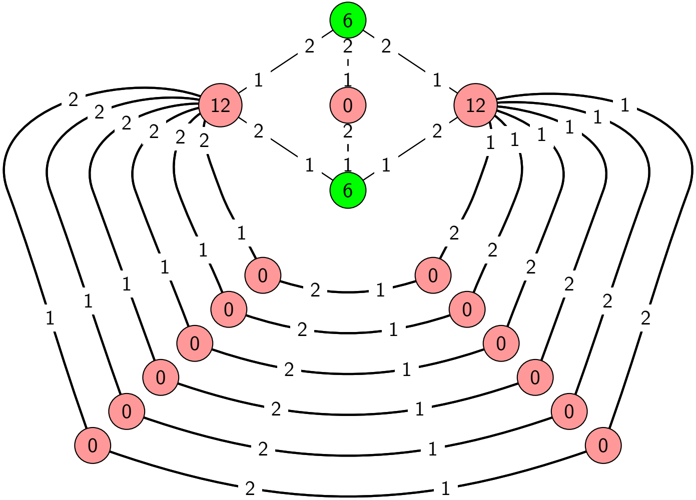
Refined idea

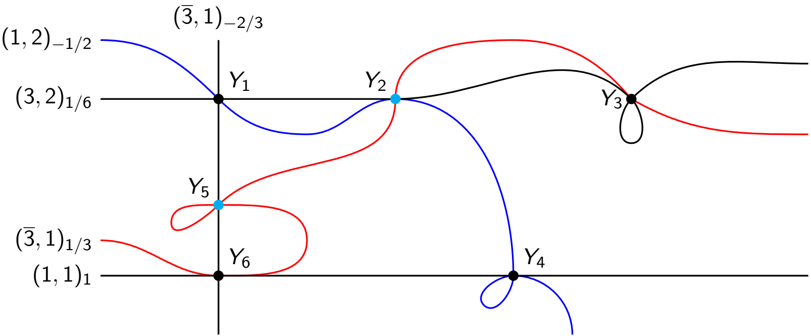
Learn about the vector-like spectra of the QSMs from root bundles on **nodal** curves.

Example: Bi-weighted graph encoding limit root

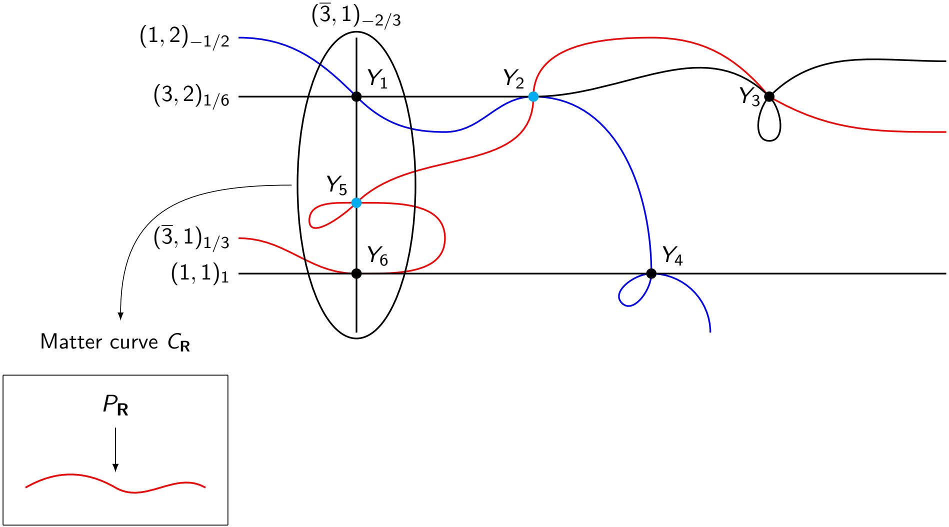


Example: Bi-weighted graph encoding limit root





$(\overline{3}, 1)_{-2/3}$
$(1, 2)_{-1/2}$
$(3, 2)_{1/6}$
$(\overline{3}, 1)_{1/3}$
$(1, 1)_1$
Y_1
Y_2
Y_3
Y_4
Y_5
Y_6



$(\overline{3}, 1)_{-2/3}$
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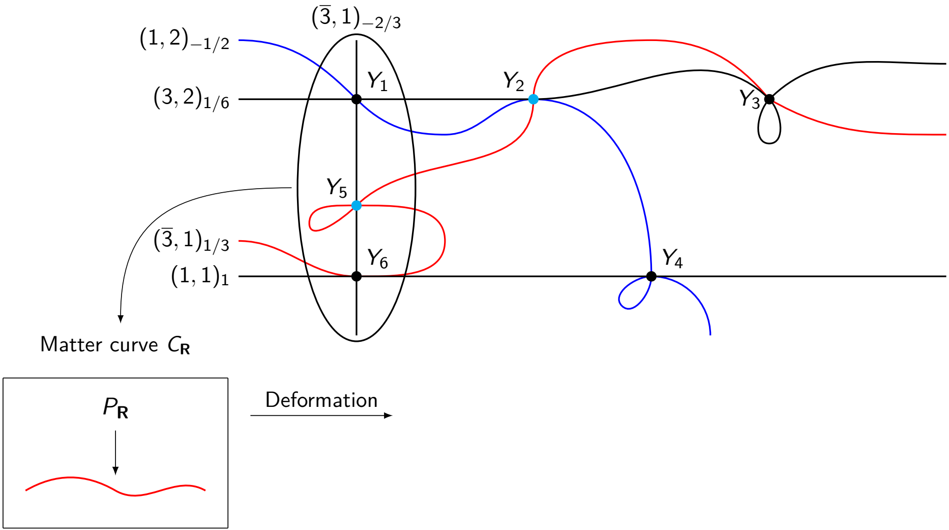
$(3, 2)_{1/6}$
Y_1
Y_2
Y_3
Y_5
$(\overline{3}, 1)_{1/3}$
$(1, 1)_1$
Y_6
Y_4
Matter curve $C_{\mathbf{R}}$
$P_{\mathbf{R}}$



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$(3,2)_{1/6}$
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$(1,1)_{1}$
Y_1
Y_2
Y_3
Y_4
Y_5
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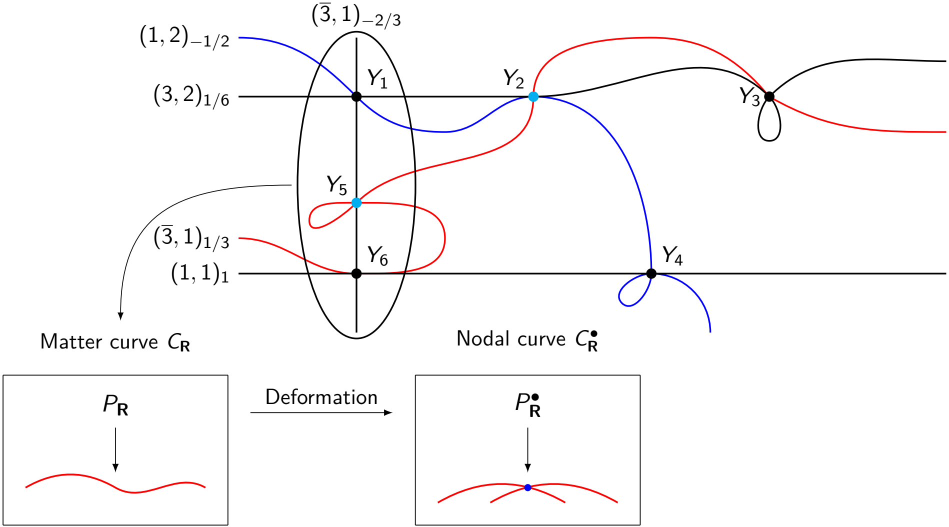
Matter curve $C_{\mathbf{R}}$
$P_{\mathbf{R}}$
Deformation



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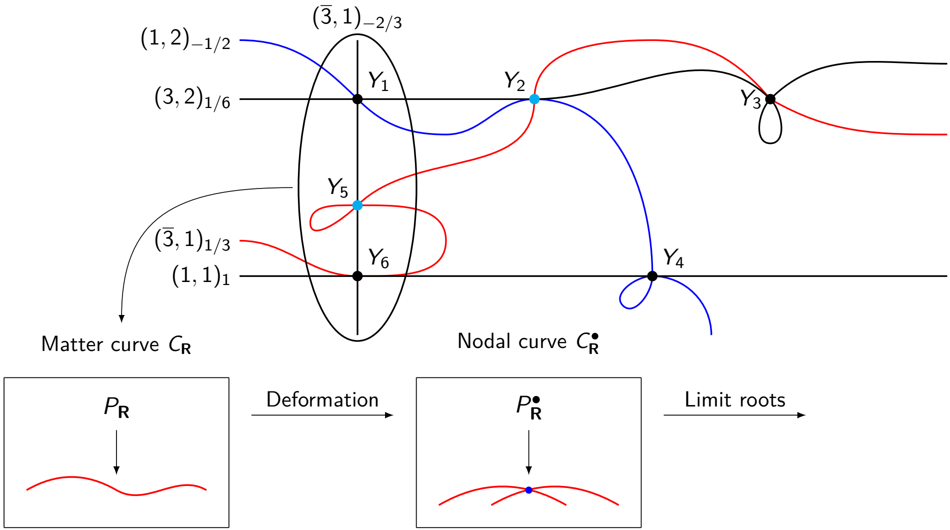
Matter curve $C_{\mathbf{R}}$
Nodal curve $C^{\bullet}_{\mathbf{R}}$
$P_{\mathbf{R}}$
Deformation
$P^{\bullet}_{\mathbf{R}}$



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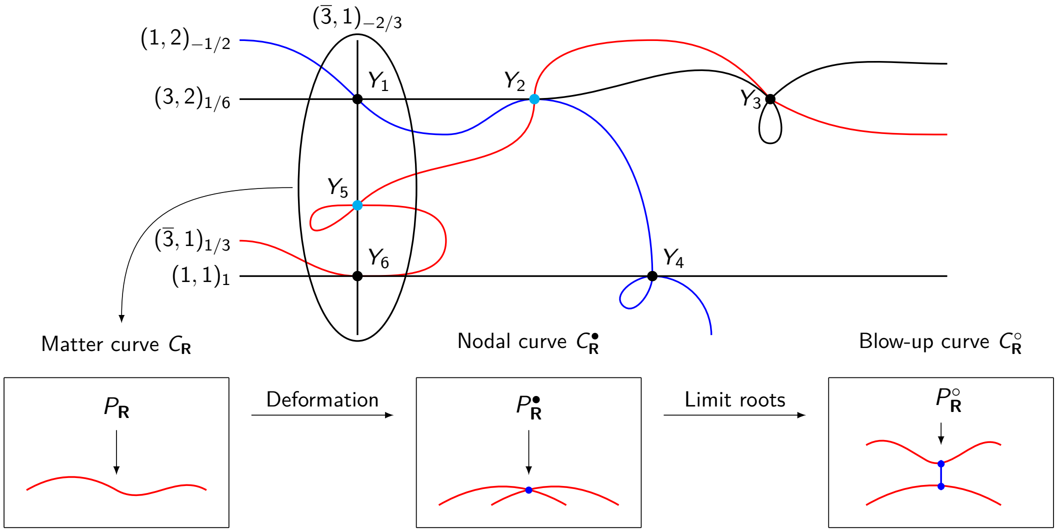
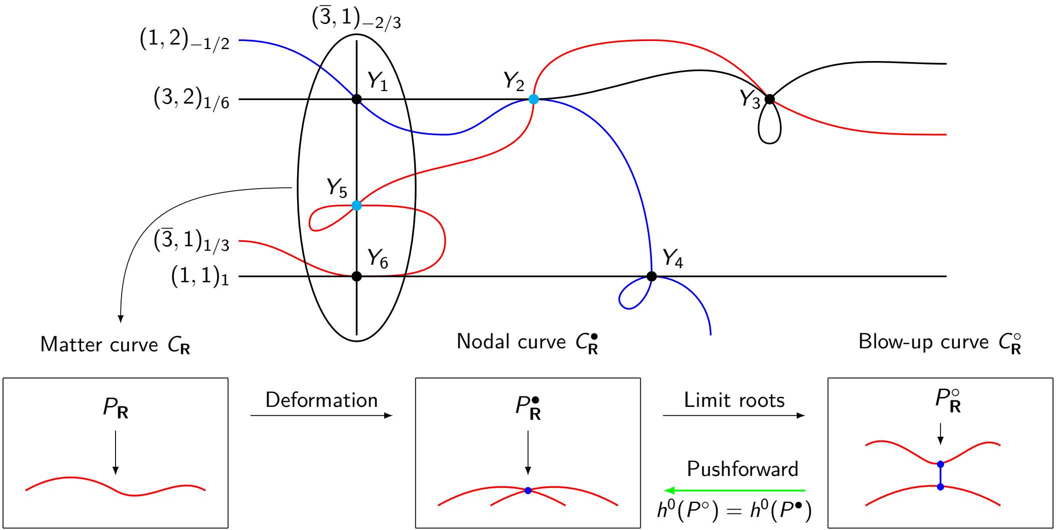
Matter curve $C_{\mathbf{R}}$
Nodal curve $C^\bullet_{\mathbf{R}}$
$P_{\mathbf{R}}$
Deformation
$P^\bullet_{\mathbf{R}}$
Limit roots

Philosophy: Local, bottom-up . . . [M.B. Cvetič Donagi Liu Ong '21], [M.B. Cvetič Liu '21], [M.B. Cvetič Donagi Ong '22]





$(\overline{3},1)_{-2/3}$
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$(3,2)_{1/6}$
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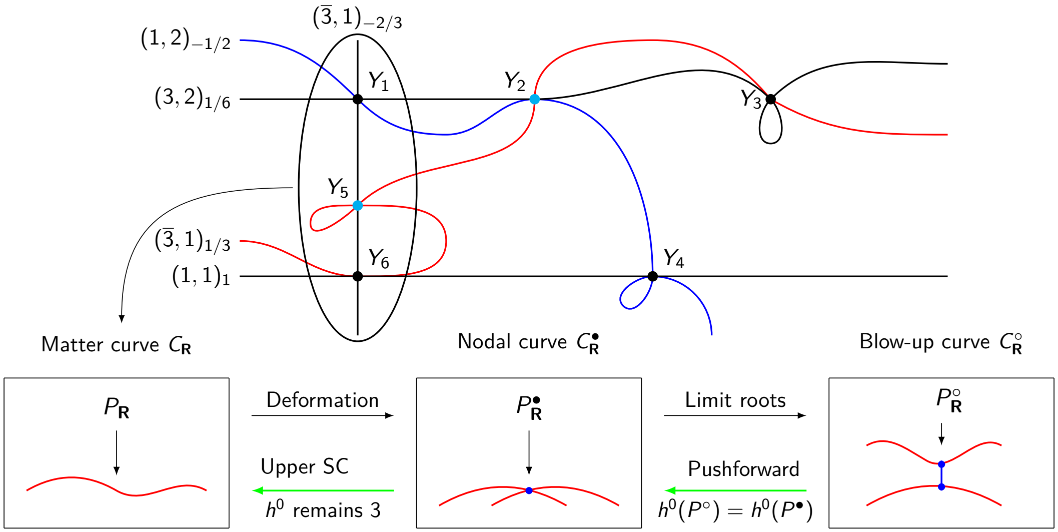
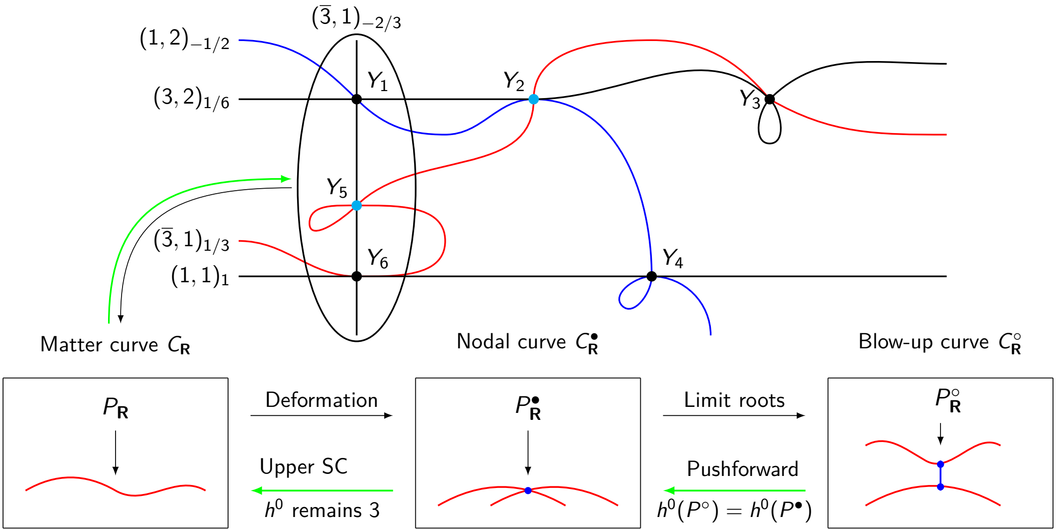
$(1,1)_1$
Y_1
Y_2
Y_3
Y_4
Y_5
Y_6
Matter curve $C_{\mathbf{R}}$
Nodal curve $C^\bullet_{\mathbf{R}}$
Blow-up curve $C^\circ_{\mathbf{R}}$
$P_{\mathbf{R}}$
Deformation
$P^\bullet_{\mathbf{R}}$
Limit roots
Pushforward
$h^0(P^\circ) = h^0(P^\bullet)$
$P^\circ_{\mathbf{R}}$

Philosophy: Local, bottom-up . . . [M.B. Cvetič Donagi Liu Ong '21], [M.B. Cvetič Liu '21], [M.B. Cvetič Donagi Ong '22]



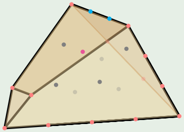
Philosophy: Local, bottom-up . . . [M.B. Cvetič Donagi Liu Ong '21], [M.B. Cvetič Liu '21], [M.B. Cvetič Donagi Ong '22]



Philosophy: Local, bottom-up and FRST invariant

[M.B. Cvetič Donagi Liu Ong '21], [M.B. Cvetič Liu '21], [M.B. Cvetič Donagi Ong '22]

Advantage: Triangulation invariant estimate of VL spectra for huge families of QSMs



Δ° ⟶ fine regular star triangulations

Family $B_3(\Delta^\circ)$ of toric F-theory base 3-folds

- - -▸

Same nodal matter curve $C_{\mathbf{R}}^{\bullet}$ $\forall X_\Sigma \in B_3(\Delta^\circ)$

[Klevers Peña Oehlmann Piragua Reuter '14], [Cvetič Klevers Peña Oehlmann Reuter '15], [Cvetič Lin Liu Oehlmann '18], [Cvetič Halverson Lin Liu Tian '19], . . .

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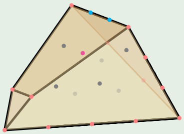

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Interlude: Computer algebra systems

- Triangulations in [M.B. Cvetič Donagi Ong '22] done with the modern computer algebra system `OSCAR`, which – due to the use of the `Julia` programming language – is expected to be very performant.
- For *fast* triangulations, also look at `CY-Tools` [Liam McAllister group], which hopefully can be available via `OSCAR` soon.

Towards “good” physical roots

(Naive) Brill-Noether theory for root bundles

Discriminate the r^{2g} line bundles $\mathcal{L} \in \mathrm{Pic}(C)$ with $\mathcal{L}^r = T$ according to $h^0(C, \mathcal{L})$:

$$r^{2g} = N_0 + N_1 + N_2 + \ldots \,, \tag{1}$$

where N_i is the number of those root bundles $\mathcal{L}$ with $h^0(C, \mathcal{L}) = i$.

Towards "good" physical roots

(Naive) Brill-Noether theory for **root bundles**

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where N_i is the number of those root bundles $\mathcal{L}$ with $h^0(C, \mathcal{L}) = i$.

Current standing

- Systematic answer unknown (to my knowledge).
- For sufficiently simple setups can count N_i, **but**:
  - Ignorance: Currently, we can sometimes only compute a lower bound to h^0.
  - Jumping circuits: h^0 can jump if nodes are specially aligned. [M.B. Cvetič Donagi Ong '22]
  - $\Rightarrow$ Denote the number of these cases by $\widetilde{N}_{\geq i}$.

$$r^{2g} = \left(\widetilde{N}_0 + \widetilde{N}_{\geq 0}\right) + \left(\widetilde{N}_1 + \widetilde{N}_{\geq 1}\right) + \ldots \,. \tag{2}$$

Brill-Noether numbers of $(\overline{\mathbf{3}}, \mathbf{2})_{1/6}$ in QSMs

- First estimates computed in [M.B. Cvetič Liu '21]:
  - count **"simple"** root bundles with minimal h^0,
  - no estimate for $\widetilde{N}_{\geq i}$.
- Refinements/extensions in [M.B. Cvetič Donagi Ong '22]:
  - count **all** root bundles,
  - discriminate via line bundle cohomology on rational tree-like nodal curves,

QSM-family (KS polytope)	# FRSTs	$h^0 = 3$	$h^0 \geq 3$	$h^0 = 4$	$h^0 \geq 4$
Δ_8°	$\sim 10^{15}$	57.3%	?	?	?
Δ_4°	$\sim 10^{11}$	53.6%	?	?	?
Δ_{134}°	$\sim 10^{10}$	48.7%	?	?	?
$\Delta_{128}^\circ, \Delta_{130}^\circ, \Delta_{136}^\circ, \Delta_{236}^\circ$	$\sim 10^{11}$	42.0%	?	?	?

Brill-Noether numbers of $(\overline{\mathbf{3}}, \mathbf{2})_{1/6}$ in QSMs

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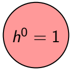
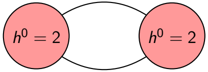
QSM-family (KS polytope)	# FRSTs	$h^0 = 3$	$h^0 \geq 3$	$h^0 = 4$	$h^0 \geq 4$
Δ_8°	$\sim 10^{15}$	76.4%	23.6%		
Δ_4°	$\sim 10^{11}$	99.0%	1.0%		
Δ_{134}°	$\sim 10^{10}$	99.8%	0.2%		
$\Delta_{128}^\circ, \Delta_{130}^\circ, \Delta_{136}^\circ, \Delta_{236}^\circ$	$\sim 10^{11}$	99.9%	0.1%		

- Stationary circuits with $h^0 = 3$:





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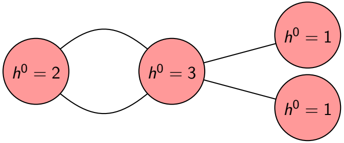
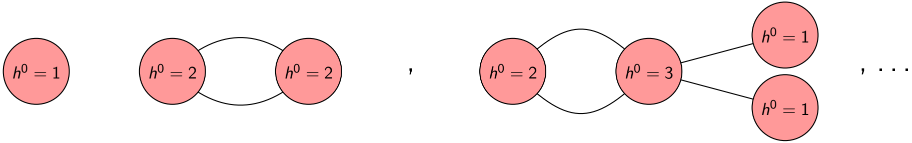


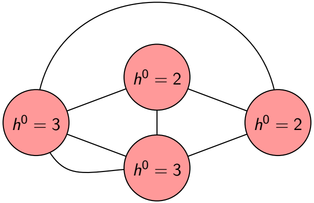
, . . .

Can we do better for $B_3(\Delta_4^\circ)$? The 1% contains . . .

- Stationary circuits with $h^0 = 3$:

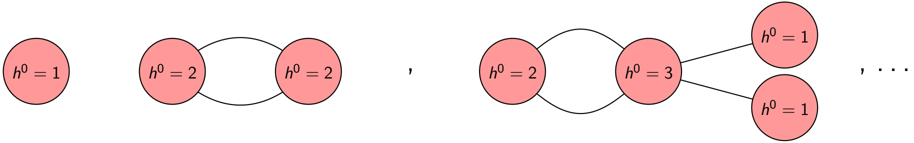


- Jumping circuit with $h^0 = 4$:

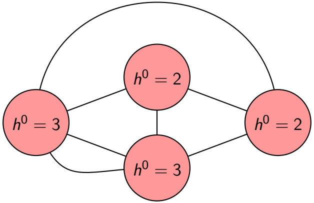


Can we do better for $B_3(\Delta_4^\circ)$? The 1% contains . . .

- Stationary circuits with $h^0 = 3$:



- Jumping circuit with $h^0 = 4$:



Mistake in first preprint [M.B. Cvetič Donagi Ong '22]

- We **wrongly** computed h^0 for the jumping circuit. Correction on the `ArXiV`.

$\Rightarrow$ $B_3(\Delta_4^\circ)$: 99.995% of solutions to **necessary** root bundle constraint have $h^0 = 3$.

Brill-Noether numbers of $(\overline{3}, 2)_{1/6}$ in QSMs [M.B. Cvetič Donagi Ong '22]

QSM-family (polytope)	$h^0 = 3$	$h^0 \geq 3$	$h^0 = 4$	$h^0 \geq 4$	$h^0 = 5$	$h^0 \geq 5$	$h^0 = 6$	$h^0 \geq 6$
Δ°_{88}	74.9	22.1	2.5	0.5	0.0	0.0		
Δ°_{110}	82.4	14.1	3.1	0.4	0.0			
Δ°_{272}, Δ°_{274}	78.1	18.0	3.4	0.5	0.0	0.0		
Δ°_{387}	73.8	21.9	3.5	0.7	0.0	0.0		
Δ°_{798}, Δ°_{808}, Δ°_{810}, Δ°_{812}	77.0	17.9	4.4	0.7	0.0	0.0		
Δ°_{254}	95.9	0.5	3.5	0.0	0.0	0.0		
Δ°_{52}	95.3	0.7	3.9	0.0	0.0	0.0		
Δ°_{302}	95.9	0.5	3.5	0.0	0.0			
Δ°_{786}	94.8	0.3	4.8	0.0	0.0	0.0		
Δ°_{762}	94.8	0.3	4.9	0.0	0.0	0.0		
Δ°_{417}	94.8	0.3	4.8	0.0	0.0	0.0	0.0	
Δ°_{838}	94.7	0.3	5.0	0.0	0.0	0.0		
Δ°_{782}	94.6	0.3	5.0	0.0	0.0	0.0		
Δ°_{377}, Δ°_{499}, Δ°_{503}	93.4	0.2	6.2	0.0	0.1	0.0		
Δ°_{1348}	93.7	0.0	6.2	0.0	0.1		0.0	
Δ°_{882}, Δ°_{856}	93.4	0.3	6.2	0.0	0.1	0.0	0.0	
Δ°_{1340}	92.3	0.0	7.6	0.0	0.1		0.0	
Δ°_{1879}	92.3	0.0	7.5	0.0	0.1		0.0	
Δ°_{1384}	90.9	0.0	8.9	0.0	0.2		0.0	

Summary and outlook

- **Statistical observation** (cf. [talk by W. Taylor]):
  In QSMs, absence of vector-like exotics in $(\overline{\mathbf{3}}, \mathbf{2})_{1/6}$, $(\overline{\mathbf{3}}, \mathbf{1})_{-2/3}$, $(\mathbf{1}, \mathbf{1})_1$ likely,
  but . . .
  - **Sufficient** condition for quantization of G_4-flux? [Jefferson Taylor Turner '21].
  - F-theory gauge potential
    - may select (proper) subset of these root bundles,
    - lead to correlated choices on distinct matter curves.
- Vector-like spectra on $C_{\mathbf{R}}^{\bullet}$ "upper bound" to those on $C_{\mathbf{R}}$.
  $\leftrightarrow$ Understand "drops" from **Yukawa interactions**? [Cvetič Lin Liu Zhang Zoccarato '19]
  $\rightarrow$ Towards the Higgs . . .
- Computationally, Higgs curve currently too challenging.
  - Need **Brill-Noether theory for root bundles on nodal curves**.
    Map from (dual) graphs (and a couple of integers) to Brill-Noether numbers.
    $\leftrightarrow$ Arena for **machine learning**? [W.i.p. with R. Hochwert]
- **Probability/statistics** for F-theory setups to arise **without vector-like exotics**.

Thank you for your attention!