FTheoryTools
A computer tool for singular elliptic fibrations

M. Bies[1], A. P. Turner[2] and the OSCAR collaboration (StringMath 2023, Melbourne)

[1] RPTU Kaiserslautern-Landau, Germany [2] University of Pennsylvania, USA

Rheinland-Pfälzische Technische Universität Kaiserslautern Landau

1 Motivation

1.1 F-theory

- Powerful tool for exploring the string landscape. See [1] for background.
- D7-brane backreaction in IIB string theory $\leftrightarrow$ **Elliptic fibration** Y.
- Non-trivial physics requires **singular** Y.
- Typically, we read-off physics from **crepant** resolution $\widehat{Y}$:

Physics	Geometry of $\widehat{Y}$
Non-Abelian gauge algebras	Intersection theory
$U(1)$ gauge factors	Mordell–Weil group
Global structure of gauge group	Mordell–Weil group
Matter curves, Yukawa points	Intersection theory
Chiral matter	G_4-flux s.t. $G_4 + \frac{1}{2}c_2(\widehat{Y}_4) \in H^{2,2}(\widehat{Y}, \mathbb{Z})$
Vector-like matter	Deligne cohomology, root bundles

1.2 Complexity of computations obstructs progress

- Imposes **large computational overhead** for analyzing models.
- Results in **duplicated effort**.
- Makes it harder for newcomers to enter the field.

2 What is `FTheoryTools`?

- **In development** component of software project `OSCAR`.
- Aims for **convenient computer tool** to simplify F-theory studies.
- Current features for singular elliptic fibration $Y \twoheadrightarrow B$:
  - Fixed base space B and (arbitrary) family of base spaces.
  - Weierstrass, Tate and hypersurface models of Y.
  - (Crepant) Resolution of Y.
  - Intersection theory and fiber analysis.

2.1 Architecture of `OSCAR`

- Combines techniques from algebra, geometry, and number theory:
  1. `Antic` (number theory)
  2. `GAP` (computational discrete algebra)
  3. `Polymake` (polyhedral geometry)
  4. `Singular` (algebraic geometry)
- Written in (**fast** programming language) `Julia`.

2.2 Further reading

- General information: **https://www.oscar-system.org**
- Tutorials: **https://www.oscar-system.org/tutorials/**
- Support for toric geometry in `OSCAR` [2].

3 U(1)-restricted SU(5) Tate model [3]

- Based on $Y_4 \twoheadrightarrow B_3$ with arbitrary B_3 and SU(5) singularity over $V(w)$.
- Defined as global Tate model with hypersurface equation in $\mathbb{P}^{2,3,1}_{[x:y:z]}$:

$$y^2 + a_{1,0}xyz + a_{3,2}w^2yz^2 = x^3 + a_{2,1}wx^2z^2 + a_{4,3}w^3xz^4\,,$$
$$a_{i,j} \cdot w^j \in H^0\left(B_3, \mathcal{O}_{B_3}\left(i \cdot \overline{K}_{B_3}\right)\right)\,.$$

- We construct this model with `FTheoryTools` as follows:

```
R,(a10,a21,a32,a43,w) = QQ["a10","a21","a32","a43","w"];
tate_sections = [w^0*a10,w^1*a21,w^2*a32,w^3*a43,R(0)];
t = global_tate_model(tate_sections,R,3)
```

- Interactively, we find the singular loci:

```
julia> singular_loci(t)[1]
(ideal(a1^5*a32*a43 - ..., (0, 0, 1), "I_1")

julia> singular_loci(t)[2]
(ideal(w), (0, 0, 5), "Split I_5")
```

The classification of singular fibers is inspired by [4, 5].

4 Literature models

- Models **often revisited** to gain insights from latest mathematics.
- Much information about F-theory models available in literature:
  - Different presentations (Weierstrass, global Tate, hypersurface, . . .).
  - Known generating sections, resolutions and physical data.
- Example: Crepant resolution for $U(1)$-restricted $SU(5)$ model in [3]:

```
julia> blowups = [[6,7,5], [2,3,1], [3,4], [2,4]];

julia> resolution = blowup_sequence(t, blowups);

julia> proper_transform = resolution[1]
ideal(-b_4_1*e_3*b_2_1*a1*z + ...)

julia> exceptionals = resolution[2]
ideal(x, y, w, e_1)
```

- `FTheoryTools` **includes database of literature models**:
  - Search by arXiv number, DOI, equation number, . . .
  - Contains as much known data as possible.

5 Revisiting U(1)-restricted SU(5) Tate model [3]

Construct and resolve this model conveniently as literature model:

```
julia> t=literature_model(arxiv_id="1109.3454",
                                              equation="3.1")
... SU(5)xU(1) restricted Tate model...

julia> resolve(t, 1)
Scheme of a toric variety with fan spanned by ...
```

6 Outlook

- Grow literature model data base.
- Provide functionality for G_4-flux and Mordell–Weil group.
- Use known computational techniques, e.g. pushforward intersections [6].
- Extend resolution techniques:
  - Support weighted blowups [7].
  - Find and implement algorithm for crepant desingularization.
- Support CICY models and yet more general scheme models.

References

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[4] K. Kodaira, *On Compact Analytic Surfaces: II*, *Annals of Mathematics* **77** (1963) 563.

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