The Standard Model From String Theory

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Outline

1. Brief introduction to string theory
   - Why String theory?
   - What is string theory?

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2. Intersecting D6-Brane Models
   - Factorisable D6-branes
   - Standard model particles in intersecting D6-branes
   - A concrete example
   - Orientifold Models

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   - Factorisable D6-branes
   - Standard model particles in intersecting D6-branes
   - A concrete example
   - Orientifold Models
3. Homological algebra, open strings and mirror symmetry

Section 1

Brief introduction to string theory

The Standard model gauge group

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The gauge group

- $G = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$
- Adjoint representation has dimension 12

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12 Force particles

- 8 gluons - an $\mathrm{SU}(3)$ connection
- $W^{\pm}, Z, Y$ - an $\mathrm{SU}(2) \times \mathrm{U}(1)_Y$ connection

The Standard model matter particles and the Higgs

Summary of properties ($G = SU(3) \times SU(2) \times U(1)_Y$)

Particle	Chirality	Representation of G	Q_{em}
quarks Q	L	$(3, \bar{2})_{\frac{1}{6}}$	$+\frac{2}{3}, -\frac{1}{3}$
up-quarks U	R	$(3, \bar{1})_{\frac{2}{3}}$	$\frac{2}{3}$
down-quarks D	R	$(3, \bar{1})_{-\frac{1}{3}}$	$-\frac{1}{3}$
leptons L	L	$(1, \bar{2})_{-\frac{1}{2}}$	$-1, 0$
charged leptons E	R	$(1, \bar{1})_{-1}$	-1
neutral leptons N	R	$(1, \bar{1})_{0}$	0
Higgs up H_U	✗	$(1, \bar{2})_{\frac{1}{2}}$	0
Higgs down H_D	✗	$(1, \bar{2})_{-\frac{1}{2}}$	0

Why string theory?

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Current understanding of physics

- General relativity - gravity
- Standard model - electromagnetic, weak and strong interaction

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- what is the physics of quantum gravity?
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Answer

String theory is a promising candidate to answer these questions.

The fundamental objects in string theory

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General Philosophy

Replace point particles by 1-dimensional objects.

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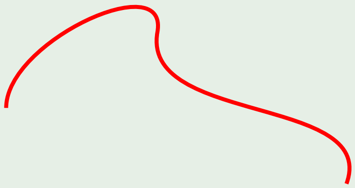
Replace point particles by 1-dimensional objects.

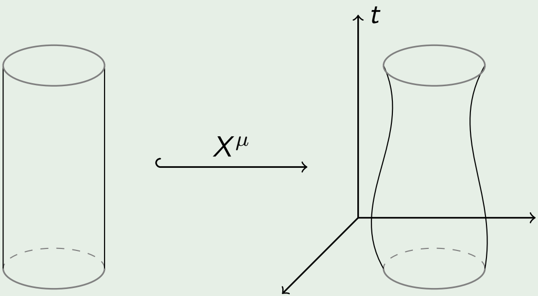
Consequence

There are two fundamental objects in string theory.

open strings	closed strings
	

Bosonic closed string CFT

Embedding of closed strings into spacetime



worldsheet (Σ, h) D-dim. spacetime $(\mathcal{S}, g)$

$$S[h, X^\mu] = -\frac{1}{4\pi\alpha'} \int_\Sigma d^2\xi \sqrt{-h}\, h^{ab}(\xi)\, \partial_a X^\mu(\xi)\, \partial_b X^\nu(\xi)\, g_{\mu\nu}(X(\xi))$$

Why is this promising?

Consistency of quantum theory

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- Conformal invariance implies:
  1. $0 = \beta^{(2)}_{\mu\nu} = \alpha' R_{\mu\nu} + \frac{1}{2} (\alpha')^2 R_{\mu\lambda\rho\sigma} R_\nu^{\lambda\rho\sigma} + \dots$
  2. ”coupling constants“ are VEVS of dynamical fields

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Standard Model + GR = string theory is promising:

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Consistency of quantum theory

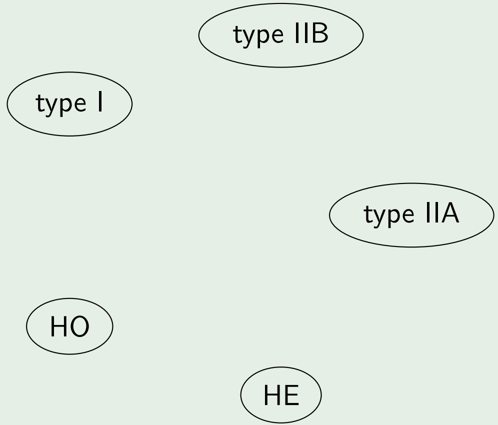
- Poincaré invariance $\Leftrightarrow$ $D = 10$
- Absence of tachyons $\Leftrightarrow$ SUSY in $D = 10$
- Conformal invariance implies:
  1. $0 = \beta_{\mu\nu}^{(2)} = \alpha' R_{\mu\nu} + \frac{1}{2}(\alpha')^2 R_{\mu\lambda\rho\sigma} R_{\nu}^{\lambda\rho\sigma} + \ldots$
  2. "coupling constants" are VEVS of dynamical fields

Standard Model + GR = string theory is promising:

- Every consistent string theory contains a graviton.
- Consistent (perturbative) theory of quantum gravity.
- No coupling constants.
- ...

Todays roadmap

M-Theory star



Todays roadmap

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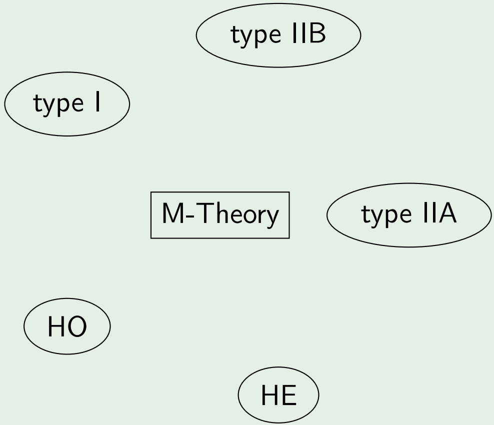

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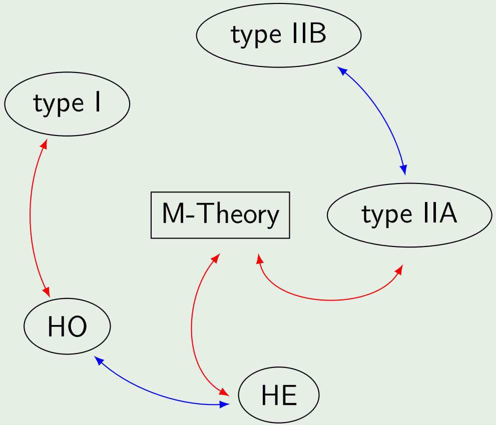

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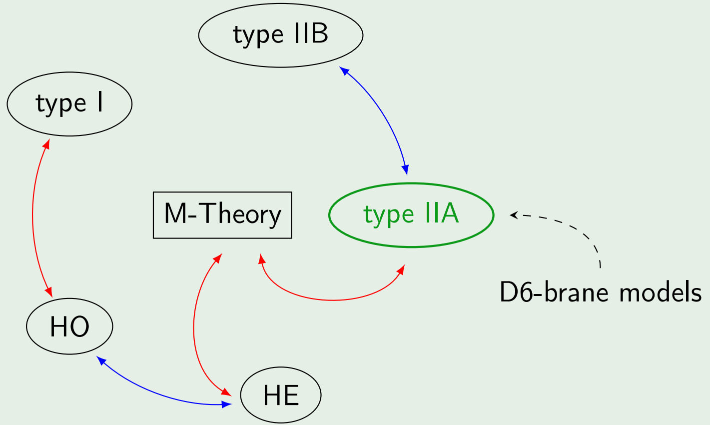

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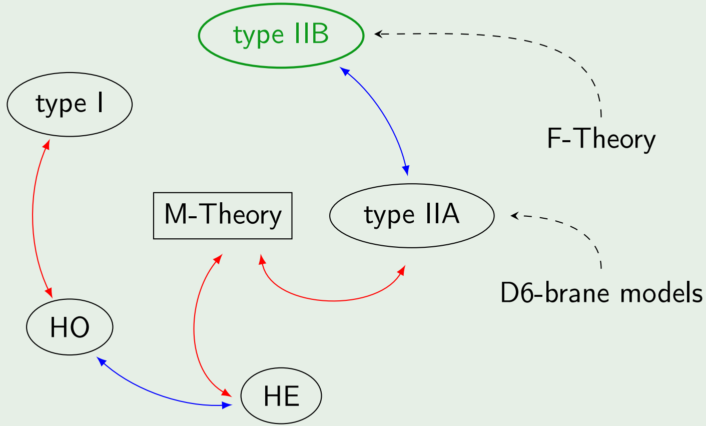
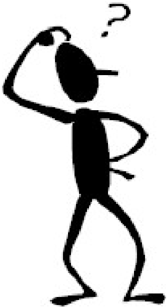


Questions?



Section 2

Intersecting D6-Brane Models

What is a D-brane?

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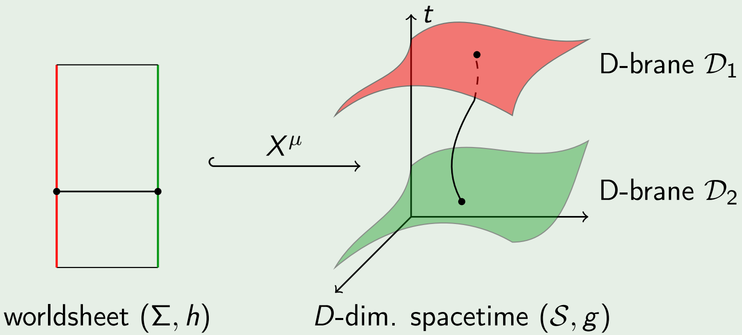
Picture of open strings and D-branes



Choice of compactification

General remarks

- In type IIA theory, D-branes always have odd dimension.
- D-brane always cover the time dimension.

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Convention

A D-brane of **dimension $\mathbf{p+1}$** is termed a **Dp-brane**.

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Choice of compactification

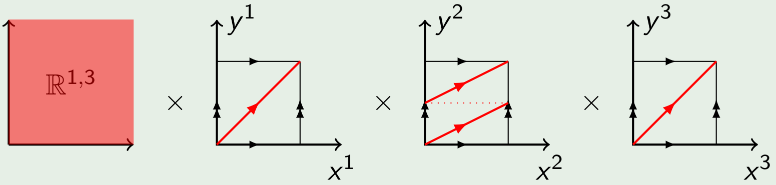
- We assume $\mathcal{S} = \mathbb{R}^{1,3} \times T^2 \times T^2 \times T^2$.
- We work with the so-called A-model in type IIA string theory.

$\Rightarrow$ Only D6-branes are present.

- We restrict further to work with **factorisable** D6-branes only.

Factorisable D6-branes

Picture of a factorisable D6-brane $\mathcal{D}$



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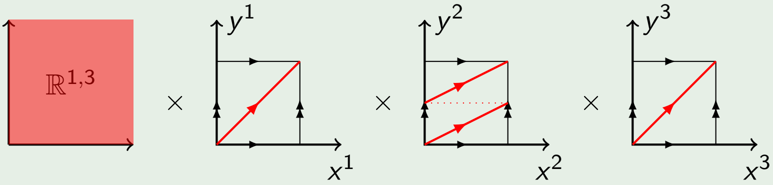

Remark

- For the time being we only care about homology.
- In this sense $\mathcal{D} \in H_1\left(T^2, \mathbb{Z}\right) \times H_1\left(T^2, \mathbb{Z}\right) \times H_1\left(T^2, \mathbb{Z}\right)$ is a factorisable D6-brane.

Factorisable D6-branes: notation

Notation

Factorisable D6-brane $\mathcal{D}_a$ denoted by

$$\mathcal{D}_a = \prod_{I=1}^{3} \left(n_a^I \left[a^I \right] + m_a^I \left[b^I \right] \right), \qquad n_a^I, m_a^I \in \mathbb{Z} \text{ coprime}$$

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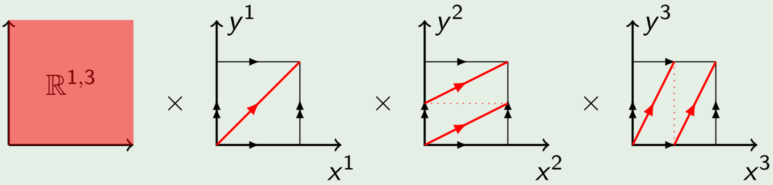
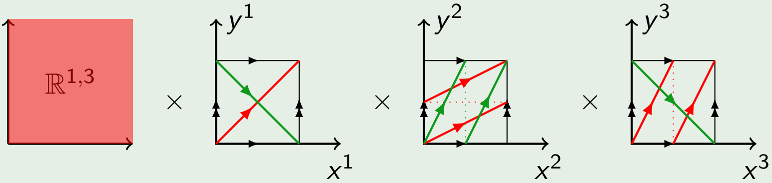
Picture of $\mathcal{D} = \left(\left[a^1\right] + \left[b^1\right] \right) \times \left(2\left[a^2\right] + \left[b^2\right] \right) \times \left(\left[a^3\right] + 2\left[b^3\right] \right)$



Factorisable D6-branes: topological intersection number

Intersections of two factorisable D6-branes $\mathcal{D}_a$ and $\mathcal{D}_b$



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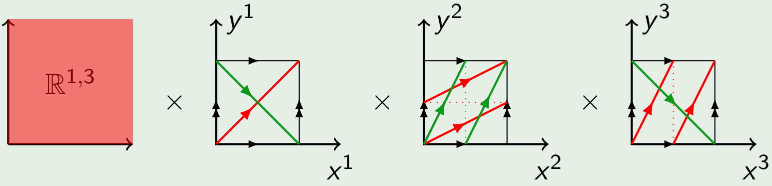

Computing topological intersection numbers

$\mathcal{D}_a = \prod_{I=1}^{3} \left(n_a^I \left[a^I \right] + m_a^I \left[b^I \right] \right)$, $\mathcal{D}_b = \prod_{I=1}^{3} \left(n_b^I \left[a^I \right] + m_b^I \left[b^I \right] \right)$

$$\Rightarrow \mathcal{D}_a \circ \mathcal{D}_b = \prod_{I=1}^{3} \left(n_a^I m_b^I - n_b^I m_a^I \right)$$

Factorisable D6-branes: family replication

Example

- $\mathcal{D}_a = (3,1) \times (1,0) \times (1,0)$
- $\mathcal{D}_b = (0,1) \times (0,1) \times (0,1)$

$$\Rightarrow \mathcal{D}_a \circ \mathcal{D}_b = \prod_{I=1}^{3} \left(n_a^I m_b^I - n_b^I m_a^I \right) = 3 \cdot 1 \cdot 1 = 3$$

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Outlook

multiple intersections $\leftrightarrow$ family replication

D-branes carry gauge theories

Fact

A stack of N-coincident D-branes carries a $U(N)$ gauge theory.

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Motivation for $N = 3$

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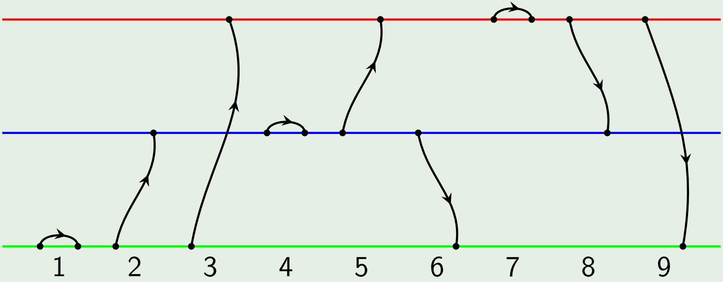

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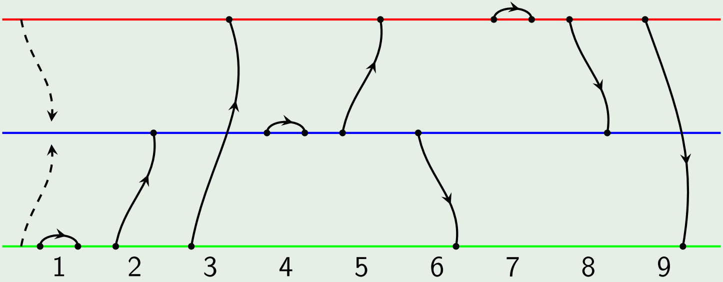
Motivation for $N = 3$

▸ Mass of strings



D-branes carry gauge theories II

Stack of N coincident branes

- N^2 strings between these branes

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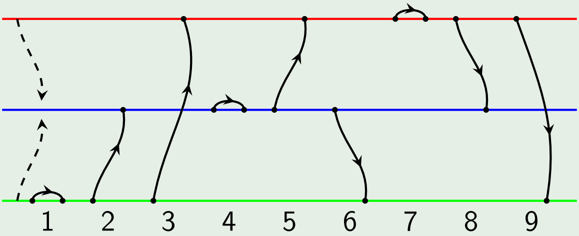

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- each gives **one** massless bosonic excitation along the stack

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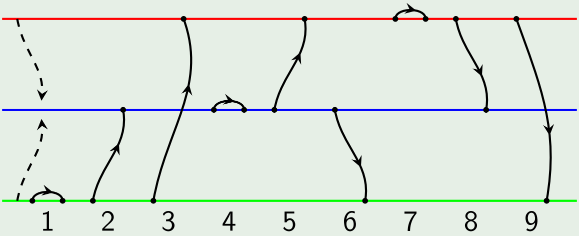

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$\Rightarrow$ Those excitations form a $\mathrm{U}(N)$ connection.

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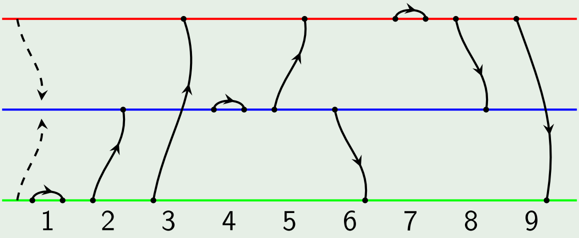

D-branes carry gauge theories II

Stack of N coincident branes

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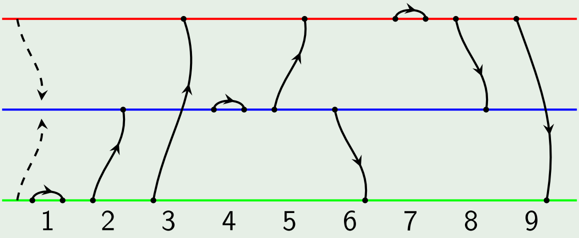
⇒ Those excitations form a $\mathrm{U}(N)$ connection.

- Structure group can be reduced to $\mathrm{SU}(N)$.

Motivation for $N = 3$



Stringy quarks

Consequence

- A string that ends on N coincident branes (and starts on another stack) is charged under $\mathrm{SU}(N) \times \mathrm{U}(1)$.

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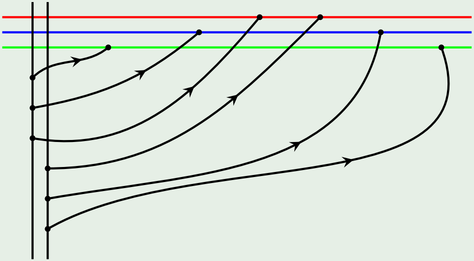
Answer: Yes! - Picture of stringy quarks



Example “A first course in string theory” by B. Zwiebach

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Wrapping numbers

Brane	$(n_a^1, m_a^1) \times (n_a^2, m_a^2) \times (n_a^3, m_a^3)$	Gauge Group
$N_1 = 3$	$(1,2) \times (1,-1) \times (1,-2)$	$SU(3) \times U(1)_1$
$N_2 = 2$	$(1,1) \times (1,-2) \times (-1,5)$	$SU(2) \times U(1)_2$
$N_3 = 1$	$(1,1) \times (1,0) \times (-1,5)$	$U(1)_3$
$N_4 = 1$	$(1,2) \times (-1,1) \times (1,1)$	$U(1)_4$
$N_5 = 1$	$(1,2) \times (-1,1) \times (2,-7)$	$U(1)_5$
$N_6 = 1$	$(1,1) \times (3,-4) \times (1,-5)$	$U(1)_6$

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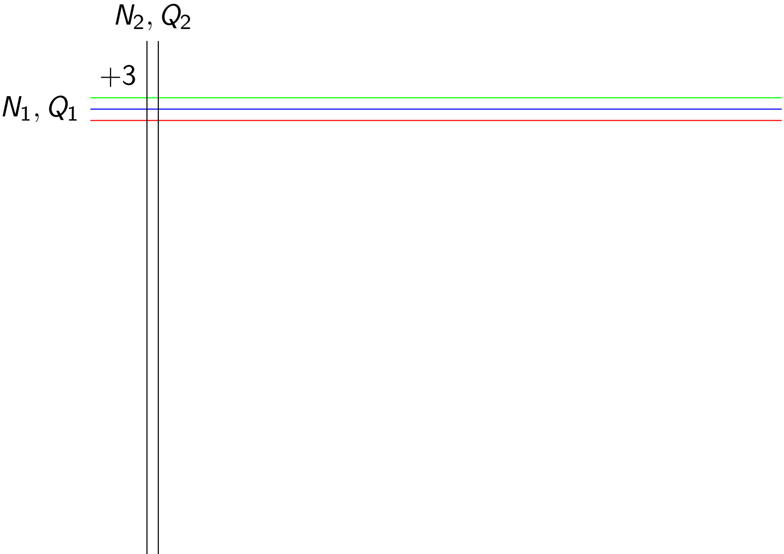

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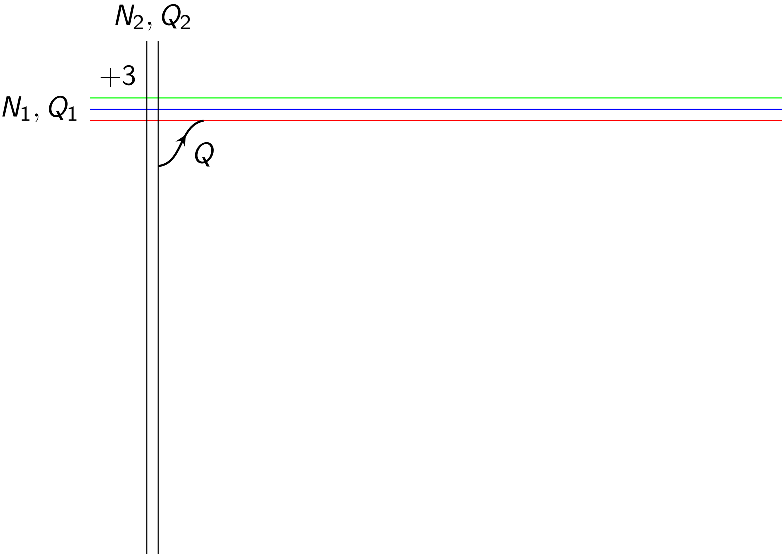

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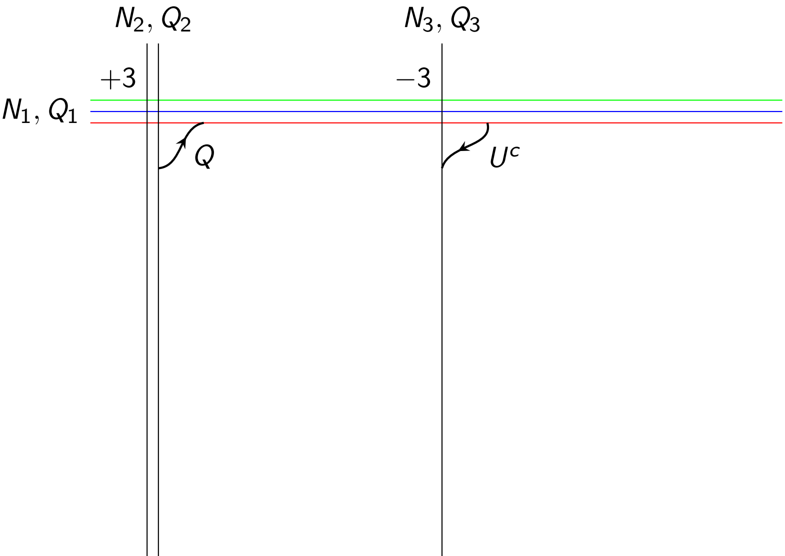

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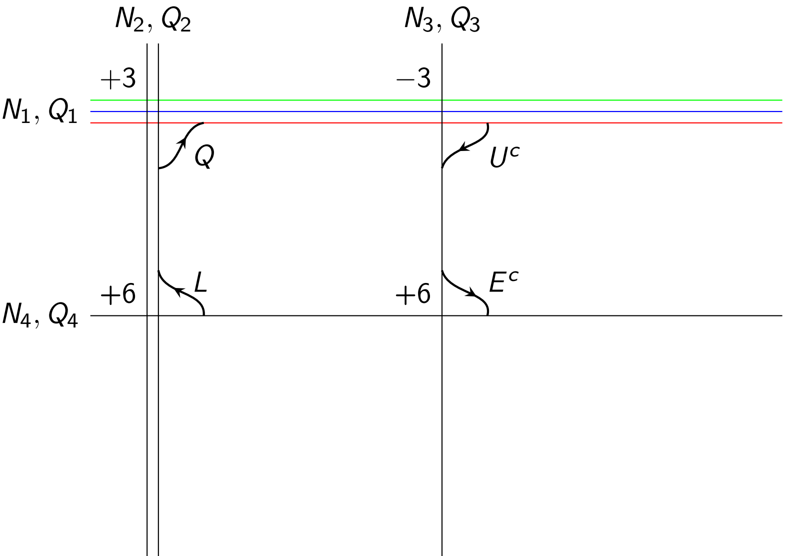

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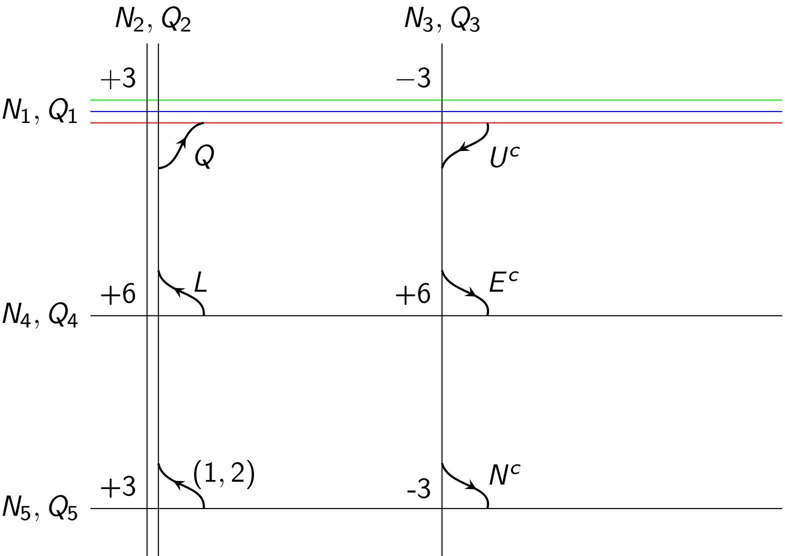

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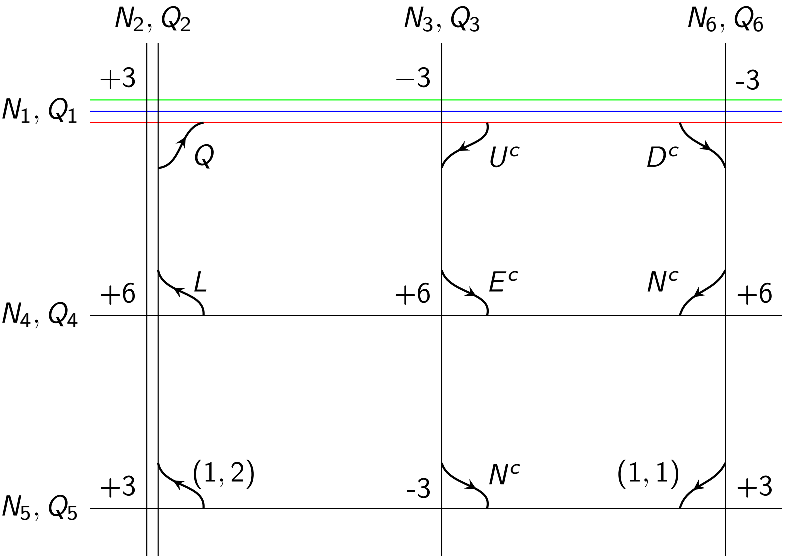

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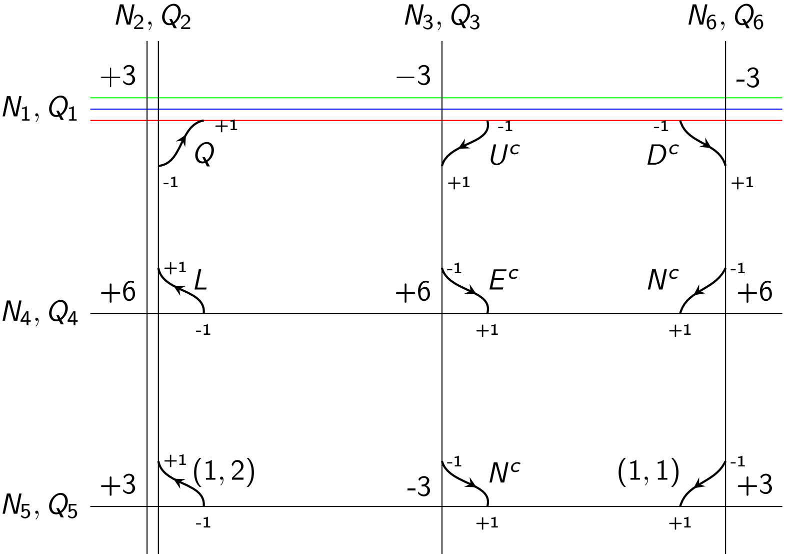
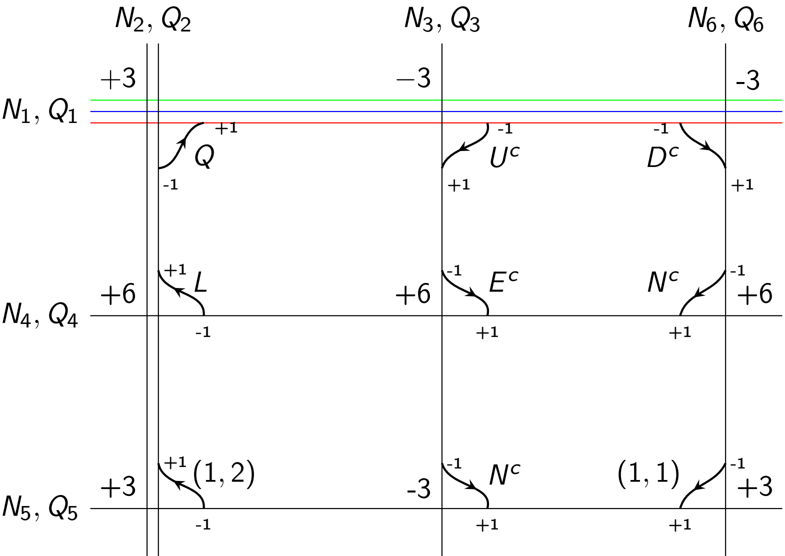


Example: $Y = -\frac{1}{3}Q_1 - \frac{1}{2}Q_2 - Q_3 - Q_5$ "A first course in ..." by B. Zwiebach



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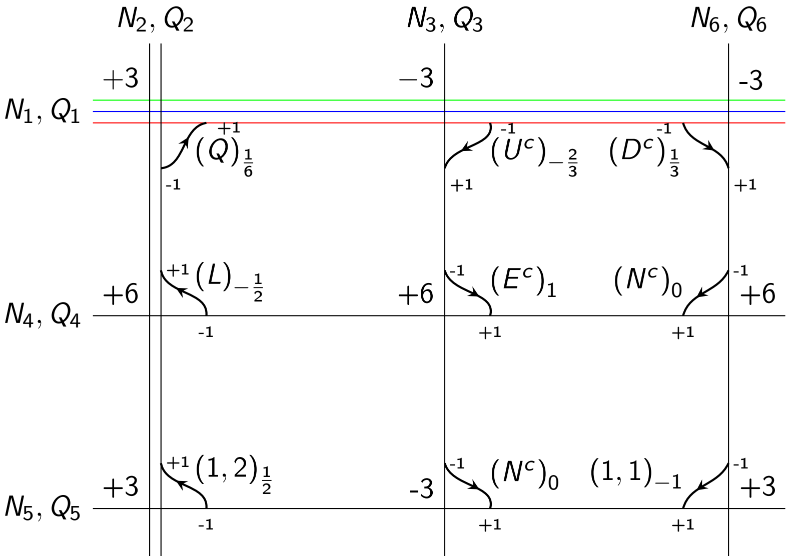

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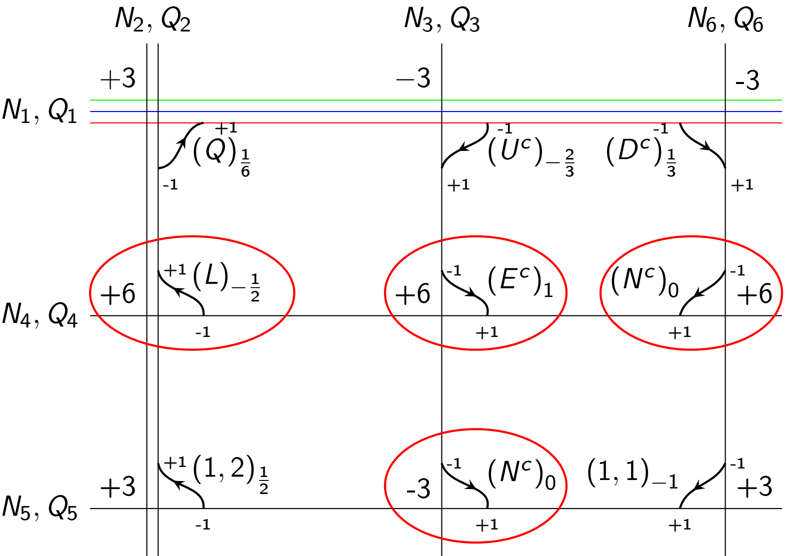

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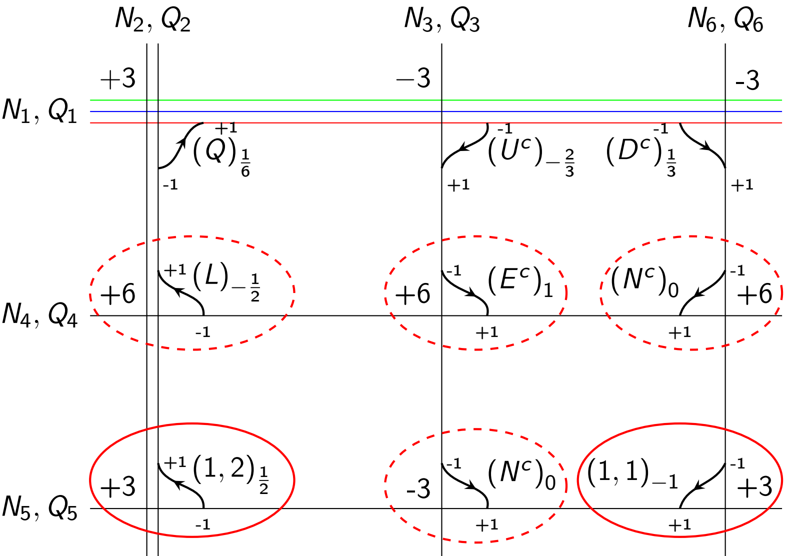

General lesson from the example

Stability

- String theory suffers from two kinds of tadpoles - R-R-tadpoles and NS-NS-tadpoles.

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Consequence

Orientifold models have to be considered, such as hep-th/0105155, hep-th/0307252, hep-th/0410134, hep-th/0502005, hep-th/0610327, hep-th/0902.3546, . . .

Need for orientifolding

Fact for D6-brane models

- Suppose we build a model that preserves (at least) $\mathcal{N} = 1$ SUSY.

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- SUSY requires orientifold plane.

Orientifolding on $T^2 \times T^2 \times T^2$

- Define involution $\overline{\sigma}$: $(z^1, z^2, z^3) \mapsto (\bar{z}^1, \bar{z}^2, \bar{z}^3)$
- Consider orientifold $\mathcal{O} := (T^2 \times T^2 \times T^2) / (\overline{\sigma} \times \Omega)$
- Fixpoint locus of $\overline{\sigma}$ is **orientifold plane O6**

A model on $\mathcal{O} = \left(T^2 \times T^2 \times T^2\right) / \left(\overline{\sigma} \times \Omega\right)$ hep-th/0105155

‣ Details on parameters

Brane	Wrapping Numbers	Gauge Group
$N_a = 3$	$\left(\frac{1}{\beta^1}, 0\right) \times \left(n_a^2, \epsilon\beta^2\right) \times \left(\frac{1}{\rho}, \frac{1}{2}\right)$	$U(3)$
$N'_a = 3$	$\left(\frac{1}{\beta^1}, 0\right) \times \left(n_a^2, -\epsilon\beta^2\right) \times \left(\frac{1}{\rho}, -\frac{1}{2}\right)$	
$N_b = 2$	$\left(n_b^1, -\epsilon\beta^1\right) \times \left(\frac{1}{\beta^2}, 0\right) \times \left(1, \frac{3\rho}{2}\right)$	$U(2)$
$N'_b = 2$	$\left(n_b^1, \epsilon\beta^1\right) \times \left(\frac{1}{\beta^2}, 0\right) \times \left(1, -\frac{3\rho}{2}\right)$	
$N_c = 1$	$\left(n_c^1, 3\rho\epsilon\beta^1\right) \times \left(\frac{1}{\beta^2}, 0\right) \times (0, 1)$	$U(1)$
$N'_c = 1$	$\left(n_c^1, -3\rho\epsilon\beta^1\right) \times \left(\frac{1}{\beta^2}, 0\right) \times (0, -1)$	
$N_d = 1$	$\left(\frac{1}{\beta^1}, 0\right) \times \left(n_d^2, -\frac{\beta^2\epsilon}{\rho}\right) \times \left(1, \frac{3\rho}{2}\right)$	$U(1)$
$N'_d = 1$	$\left(\frac{1}{\beta^1}, 0\right) \times \left(n_d^2, \frac{\beta^2\epsilon}{\rho}\right) \times \left(1, -\frac{3\rho}{2}\right)$	

A model on the orientifold $\mathcal{O}$ hep-th/0105155

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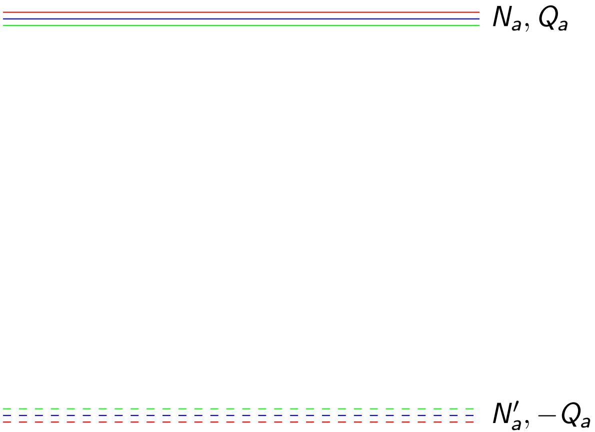
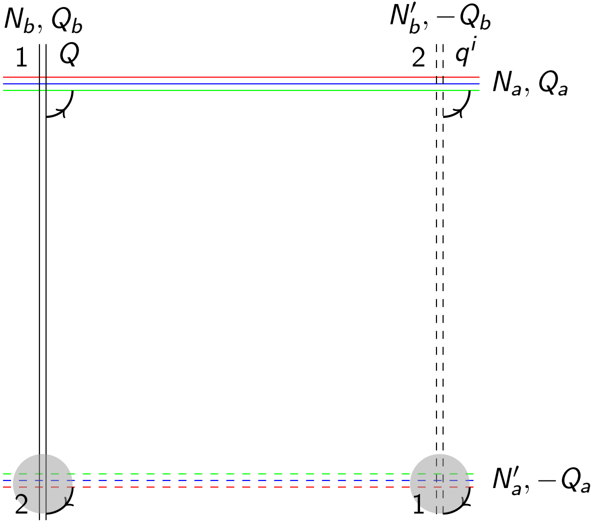


A model on the orientifold $\mathcal{O}$ hep-th/0105155



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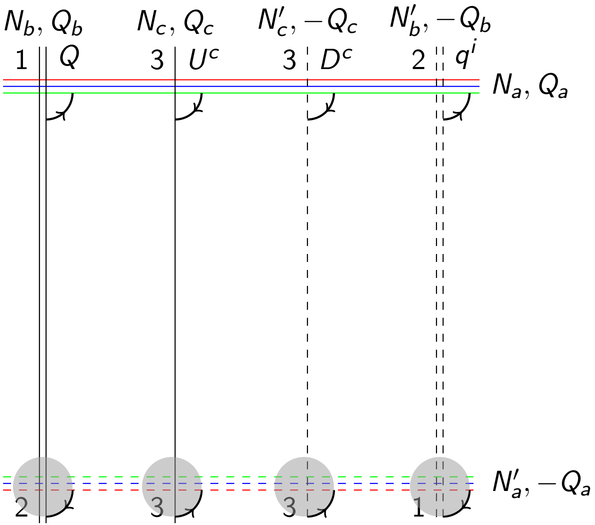

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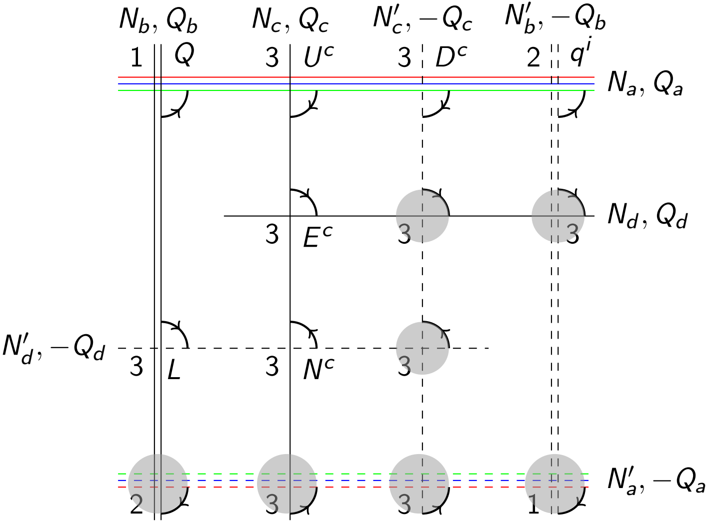

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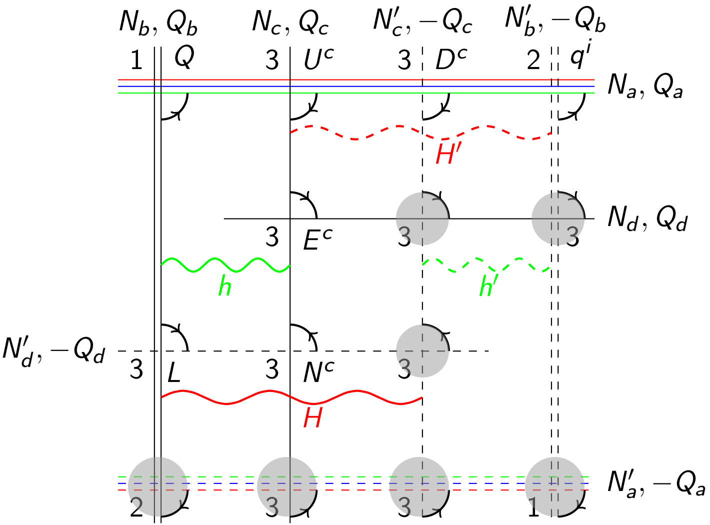
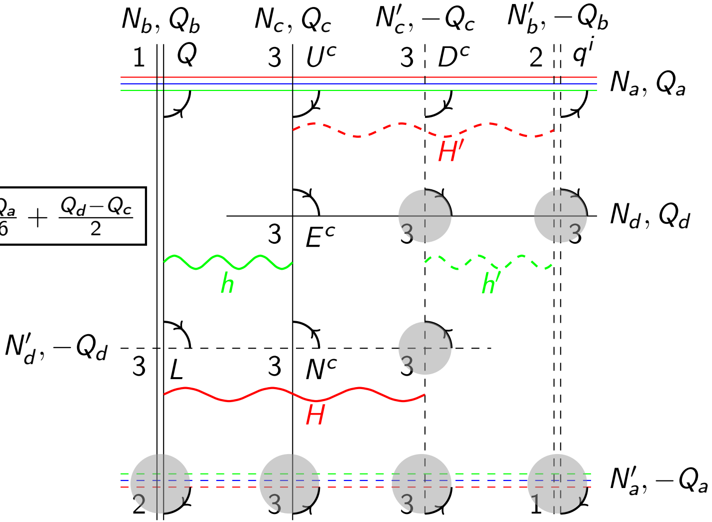


A model on the orientifold $\mathcal{O}$ hep-th/0105155

To summary on orientifold models

$$Y = \frac{Q_a}{6} + \frac{Q_d - Q_c}{2}$$



RR-Tadpole cancellation I

Cancellation of R-R tadpoles hep-th/0307252

- Find $\mathcal{D}_{O6} = 8 \prod_{I=1}^{3} \left[a^I\right]$.
- R-R tadpole cancellation then requires

$$\sum_{\text{branes } \mathcal{D}_a} N_a \left(\mathcal{D}_a + \mathcal{D}'_a\right) - 4\mathcal{D}_{O6} = [0]$$

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In terms of the wrapping numbers . . .

- $\sum_{\text{branes } \mathcal{D}_a} N_a n_a^a n_a^2 n_a^3 = 16$
- $\sum_{\text{branes } \mathcal{D}_a} N_a n_a^I m_a^J m_a^K = 0$ for $I \neq J \neq K \neq I$

RR-Tadpole cancellation II

K-Theory charges

- D-brane charge classified by K-theory hep-th/9810188, hep-th/0307252.

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$\Rightarrow$ Additional K-theory constraint of even number of $\mathrm{USp}(2, \mathbb{C})$ fundamentals needed.

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- D-brane charge classified by K-theory hep-th/9810188, hep-th/0307252.

$\Rightarrow$ Additional K-theory constraint of even number of $USp(2,\mathbb{C})$ fundamentals needed.

In terms of wrapping numbers . . .

. . . for rectangular tori in $\left(T^2 \times T^2 \times T^2\right) / \left(\sigma \times \Omega \times (-1)^{F_L}\right)$

- $\displaystyle\sum_{\text{branes } \mathcal{D}_a} N_a m_a^1 m_a^2 m_a^3 \in 2\mathbb{Z}$
- $\displaystyle\sum_{\text{branes } \mathcal{D}_a} N_a m_a^I n_a^J n_a^K \in 2\mathbb{Z}$ for $I \neq J \neq K \neq I$

Supersymmetry condition

General philosophy hep-th/9507158

- Orientifold plane $O6$ preserves some supersymmetry.
- Configuration of D6-branes preserves at least $\mathcal{N} = 1$ of this supersymmetry if each D6-brane satisfies

$$\Theta_a^1 + \Theta_a^2 + \Theta_a^3 = 0 \text{ mod } 2\pi$$

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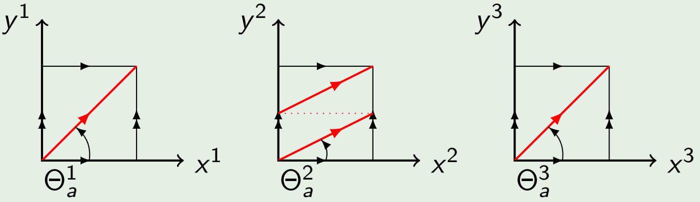
Picture of angles Θ_a^I



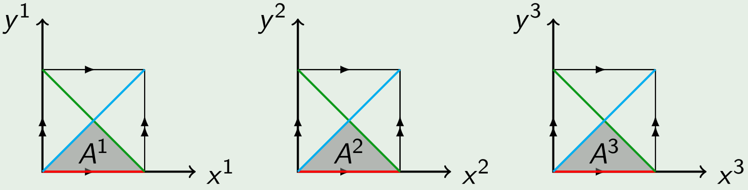
Yukawa couplings

Fact hep-th/0303083

Interaction between 2 **massless** fermions and 1 **massless** boson - all located at different intersections - is governed by

$$Y \sim \exp\left(-A^1\right) \cdot \exp\left(-A^2\right) \cdot \exp\left(-A^3\right)$$

Picture



A model on $\mathcal{O}/\left(\mathbb{Z}_2 \times \mathbb{Z}_2\right)$ hep-th/0107166, hep-th/0107143

Wrapping numbers of branes

Brane	$(n_a^1, m_a^1) \times (n_a^2, m_a^2) \times (n_a^3, \widetilde{m}_a^3)$	Gauge Group
$A_1 = 4$	$(0,1) \times (0,-1) \times \left(2, \widetilde{0}\right)$	$U(1)^2$
$A_2 = 1$	$(1,0) \times (1,0) \times \left(2, \widetilde{0}\right)$	$USp(2,\mathbb{C})_A$
$B_1 = 2$	$(1,0) \times (1,-1) \times \left(1, \widetilde{\tfrac{3}{2}}\right)$	$SU(2) \times U(1)$
$B_2 = 1$	$(1,0) \times (0,1) \times \left(0, \widetilde{-1}\right)$	$USp(2,\mathbb{C})_B$
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Wrapping numbers of image branes

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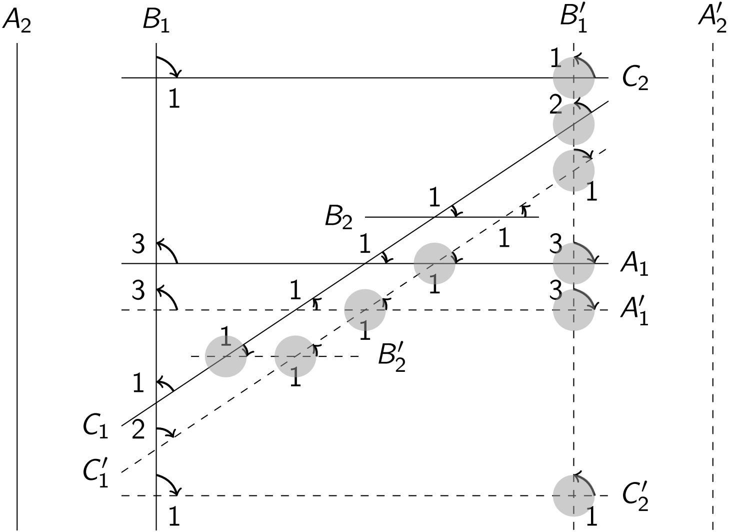

Extension of search

Back to example

Example: $\mathcal{O}/\left(\mathbb{Z}_2 \times \mathbb{Z}_2\right) = \left(T^2 \times T^2 \times T^2\right) / \left(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \overline{\sigma} \times \Omega\right)$

- 11 **semi-realistic** models constructed hep-th/0403061 but
  - ✗ matter particles are missing/ too many present
  - ✗ exotic matter present
- Systematic computer analysis was performed hep-th/0606109

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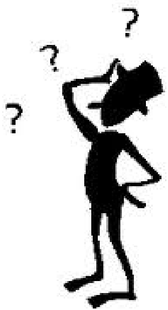
Extension of search

- Different orientifolds, e.g. hep-th/0211059, hep-th/0303015, hep-th/0309158, hep-th/0407181, hep-th/0404055, hep-th/1303.6845, . . .
  - $\left(T^2 \times T^2 \times T^2\right) / \left(\mathbb{Z}_4 \times \overline{\sigma} \times \Omega\right)$
  - $\left(T^2 \times T^2 \times T^2\right) / \left(\mathbb{Z}_2 \times \mathbb{Z}_4 \times \overline{\sigma} \times \Omega\right)$
- Magnetised D7-branes in type IIB hep-th/0702094, hep-th/0610327, hep-th/0701154, . . .

Questions?



Section 3

Homological algebra, open strings and mirror symmetry

What is an A-brane on a CY X?

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Definition: Lagrangian manifold

A submanifold $Z \subset X$ of a CY (X, J, ω, Ω) is a Lagrangian manifold if the following two conditions are satisfied:

- $\omega\big|_Z = 0$
- $\dim_{\mathbb{R}}(Z) = \frac{1}{2}\dim_{\mathbb{R}}(X)$

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Answer to the above question: hep-th/0403166

An A-brane on a CY 3-fold (X, J, ω, Ω) is an object in the Fukaya category $\mathfrak{Fuk}(X)$.

The B model

Remark hep-th/9112056

- There exists a model - called the B-model - in type IIB string theory.
- In the B-model D3-, D5-, D7- and D9-branes are present.
- Collectively they are labeled **B-branes**.

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Remark hep-th/9112056

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- In the B-model D3-, D5-, D7- and D9-branes are present.
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More precisely hep-th/0403166

A B-brane on a CY 3-fold (Y, J, ω, Ω) is an object in the category $\mathbf{D}^b(\mathfrak{Coh}(Y))$.

Consequences

Open string between B-branes hep-th/0403166 hep-th/0208104

- Consider the holomorphic vector bundles $\mathcal{E}_1$ and $\mathcal{E}_2$ as B-branes.

$\Rightarrow$ A massless string excitation from $\mathcal{E}_1$ to $\mathcal{E}_2$ of ghost number q is an element of $\mathrm{Ext}^q(\mathcal{E}_1, \mathcal{E}_2)$.

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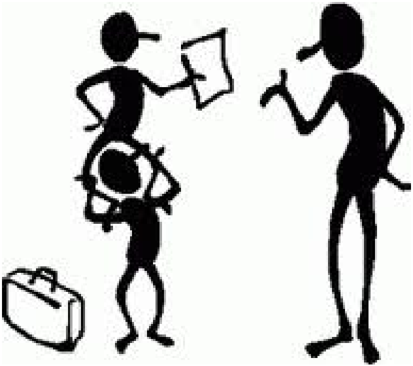
An approach to mirror symmetry hep-th/0403166

Be X, Y mirror CY-manifolds, then $\mathrm{Tr}\mathfrak{Fuk}(X) \simeq \mathbf{D}^b(\mathfrak{Coh}(Y))$.

Thank you for your attention!



Masses For Strings

Back to original frame

General formula

$$\alpha' M^2 = N_{\perp,\nu} + \frac{Y^2}{4\pi^2\alpha'} + \nu \cdot \sum_{I=1}^{3} \left| \vartheta_{ab}^{I} \right| - \nu$$

- $Y \widehat{=}$ length of string
- $\nu = \begin{cases} 0 & \text{Ramond sector} \\ \frac{1}{2} & \text{Neveu-Schwarz sector} \end{cases}$
- $\vartheta_{ab}^{I} \widehat{=} \frac{\text{intersection angle in I-th two-torus}}{\pi}$

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Example

Ground state in NS-sector has $2\alpha' M^2 = \sum_{I=1}^{3} \left|\vartheta^I_{ab}\right| - 1$

Classification of D6-Branes I

Label	(P, Q, R, S)	$\left(n_a^{1,o}, n_a^{2,o}, n_a^{3,o}\right)$	$\left(m_a^{1,o}, m_a^{2,o}, m_a^{3,o}\right)$
A1	$(-,+,+,+)$	$(+,+,-)$	$(+,+,-)$
A2	$(+,-,+,+)$	$(+,+,+)$	$(+,-,-)$
A3	$(+,+,-,+)$	$(+,+,+)$	$(-,+,-)$
A4	$(+,+,+,-)$	$(+,+,+)$	$(-,-,+)$
B1	$(+,+,0,0)$	$(1,+,+)$	$(0,+,-)$
B2	$(+,0,+,0)$	$(+,1,+)$	$(+,0,-)$
B3	$(+,0,0,+)$	$(+,+,1)$	$(+,-,0)$
B4	$(0,+,+,0)$	$(+,+,0)$	$(-,-,1)$
B5	$(0,+,0,+)$	$(+,0,+)$	$(-,1,-)$
B6	$(0,0,+,+)$	$(0,+,+)$	$(1,-,-)$

Classification of D6-Branes II

◂ Back to original frame

Label	(P, Q, R, S)	$\left(n_a^{1,o}, n_a^{2,o}, n_a^{3,o}\right)$	$\left(m_a^{1,o}, m_a^{2,o}, m_a^{3,o}\right)$
C1	$(1, 0, 0, 0)$	$(1, 1, 1)$	$(0, 0, 0)$
C2	$(0, 1, 0, 0)$	$(1, 0, 0)$	$(0, 1, -1)$
C3	$(0, 0, 1, 0)$	$(0, 1, 0)$	$(1, 0, -1)$
C4	$(0, 0, 0, 1)$	$(0, 0, 1)$	$(1, -1, 0)$

ab-sector

Definition

- Strings from π_a to π_b form **ab-sector**

ab-sector

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Properties

- $U(N_a) - U(N_b)$ bifundamentals in ab-sector
- Ramond ground state is massless, chiral fermion
- Tension forces ab-sector strings to locate at intersection

$\Rightarrow$ Propatation **only** in the external space $\mathbb{R}^{1,3}$

- multiple intersection $\pi_a \circ \pi_b = 3$ is possible

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Conclusion

- ab-sector can give rise to **matter particles**

aa-sector

Definition

- Strings from π_a to π_a form **aa-sector**

aa-sector

Definition

- Strings from π_a to π_a form **aa-sector**

Properties

- Adjoint representations of $U(N_a)$
- Neveu-Schwarz ground state is massless boson
- Location not fixed in $T^2 \times T^2 \times T^2$

$\Rightarrow$ Winding and KK-states can appear

aa-sector

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Properties

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- Neveu-Schwarz ground state is massless boson
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Conclusion

- aa-sector can give rise to Standard Model **gauge bosons**

Topological intersection number

◂ Back to original frame

Fast derivation

- Define

$$\left[a^I\right] \circ \left[b^J\right] := \delta^{IJ} =: -\left[b^J\right] \circ \left[a^I\right]$$

  All other intersections vanish.

- Then for two 3-cycles
  - $\mathcal{D}_a = \prod_{I=1}^{3} \left(n_a^I \left[a^I\right] + m_a^I \left[b^I\right]\right)$
  - $\mathcal{D}_b = \prod_{I=1}^{3} \left(n_b^I \left[a^I\right] + m_b^I \left[b^I\right]\right)$

  the topological intersection number is given by

$$\mathcal{D}_a \circ \mathcal{D}_b = \prod_{I=1}^{3} \left(n_a^I m_b^I - n_b^I m_a^I\right)$$

Closed String Quantisation I

Simplifying assumptions

- Take $\mathcal{S} = \mathbb{R}^{1,d-1}$.
- Can achieve locally $h = \eta^{ab}$ (conformal invariance).

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Strategy

- Take $h = \eta^{ab}$ and quantise the theory **locally**.
- The classical theory $S\left[h, X^{\mu}\right]$ treats h as dynamical field.

$\Rightarrow$ Implement its e.o.m **after** quantisation.

Closed String Quantisation II

Simplified action

Taking $h = \eta^{ab}$ and $g = \eta^{\mu\nu}$ gives

$$S\left[X^{\mu}\right] = \frac{T}{2}\int\limits_{\Sigma} d\tau d\sigma \left[\left(\partial_{\tau}X\right)^{2} - \left(\partial_{\sigma}X\right)^{2}\right]$$

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Classical e.o.m. and boundary condition

Look for functions $X^{\mu} : \mathbb{R}^{2} \to \mathbb{R}\ ,\ (\tau,\sigma) \to X^{\mu}(\tau,\sigma) \in L^{2}\left(\mathbb{R}^{2}\right)$ such that

- the string is closed: $X^{\mu}(\tau,\sigma = 0) = X^{\mu}(\tau,\sigma = l)$
- the e.o.m. are satisfied, i.e. $\left(\partial_{\tau}^{2} - \partial_{\sigma}^{2}\right) X^{\mu} = 0$

Closed String Quantisation III

Most general solution

$$X^\mu(\tau,\sigma) = x^\mu + \frac{2\pi\alpha'}{L} p^\mu \tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n\in\mathbb{Z}, n\neq 0} \frac{\alpha_n^\mu}{n} \cdot e^{-\frac{2\pi}{L} in(\tau-\sigma)}$$

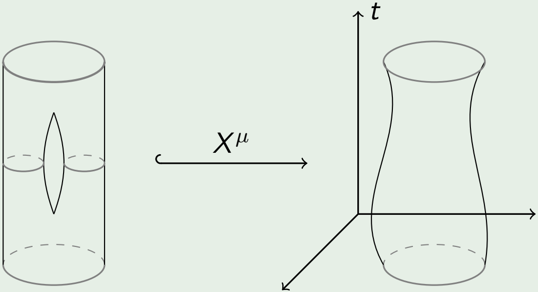
$$= + i\sqrt{\frac{\alpha'}{2}} \sum_{n\in\mathbb{Z}, n\neq 0} \frac{\tilde{\alpha}_n^\mu}{n} \cdot e^{-\frac{2\pi}{L} in(\tau+\sigma)}$$

Poisson brackets

$$\{\alpha_m^\mu, \alpha_n^\nu\} = \{\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu\} = -im\delta_{m+n,0}\eta^{\mu\nu}, \quad \{\alpha_m^\mu, \tilde{\alpha}_n^\nu\} = 0, \quad \{x^\mu, p^\nu\} = \eta^{\mu}$$

Why no coupling constants?

A closed string self-interaction



worldsheet (Σ, h) D-dim. spacetime $(\mathcal{S}, g)$

Consequence

self-interaction = **free** CFT on worldsheet Σ with **one handle**

Wrapping numbers parameters

Back to wrapping numbers

Parameter	Values
β^1	$\left\{\frac{1}{2}, 1\right\}$
β^2	$\left\{\frac{1}{2}, 1\right\}$
ϵ	$\{-1, 1\}$
ρ	$\left\{\frac{1}{3}, 1\right\}$
n_a^2	$\mathbb{Z}$
n_b^1	$\mathbb{Z}$
n_c^1	$\mathbb{Z}$
n_d^2	$\mathbb{Z}$

D-branes carry gauge theories II

Mass of an open string excitation between parallel D-branes

$$\alpha' M^2 \left|\varphi\right\rangle = \left(N + \alpha' \left(T \Delta x\right)^2 - 1\right) \left|\varphi\right\rangle$$

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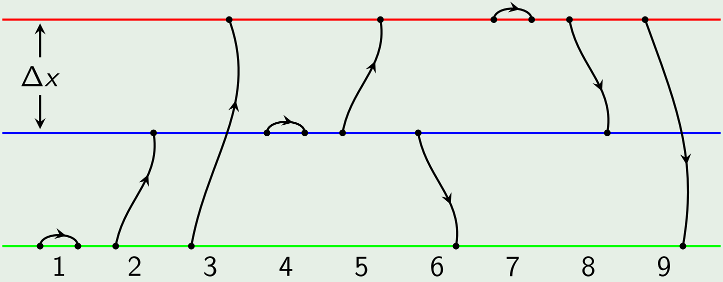
Motivation for $N = 3$



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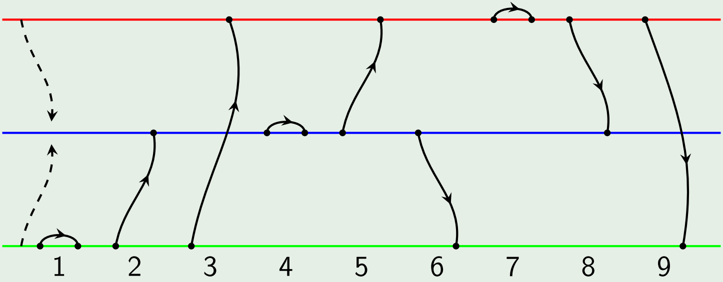

D-branes carry gauge theories III

Labels of massless bosonic string excitations along Dp-brane

- excitations **along** D_p: $(A_a)^m_n$ (i.e. $a = 0, \ldots, p$)
- excitations **normal** to D_p: $(X_i)^m_n$ (i.e. $i = p + 1, \ldots, D - 1$)

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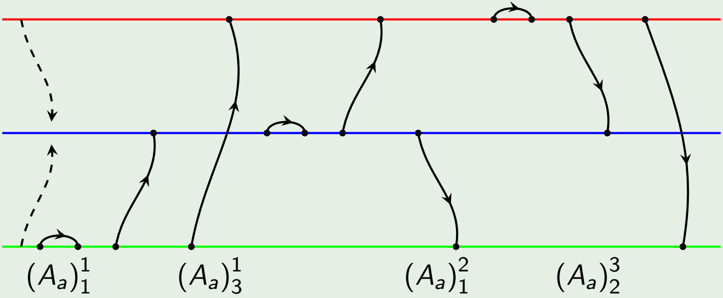

D-branes carry gauge theories IV

Fact

- $(A_a)^m_n$ form a $U(N)$ connection
- $(X_i)^m_n$ are scalar fields in the adjoint rep. of $U(N)$
- Strings that end on a stack of N-coincident Dp-branes are also charged under this $U(N)$ gauge group

D-branes carry gauge theories IV

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Question

Can we hence use strings between a $U(2)$ and a $U(3)$ brane stack to model quarks?

Supersymmetric D6-branes

Fact

A D6-brane $Z \subset X$ preserves supersymmetry iff $\mathrm{Im}\left(e^{-i\varphi}(\Omega)\right)\big|_Z = 0$.

Supersymmetric D6-branes

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A D6-brane $Z \subset X$ preserves supersymmetry iff
$\mathrm{Im}\left(e^{-i\varphi}(\Omega)\right)\big|_Z = 0.$

Consequence

supersymmetric D6-branes $\leftrightarrow$ special Lagrange manifolds $Z \subset X$

Orientifold plane as special Lagrange manifold

σ is a real structure

The antiholomorphic involution $\overline{\sigma}\colon T^6 \to T^6$ has the following properties:

- Locally it is complex conjutation.
- $\sigma^*\omega = -\omega$
- $\sigma^*\Omega = \overline{\Omega}$

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Consequence

- Fixpoint locus of $\overline{\sigma}$ defines a special Lagrange manifold - the orientifold plane $O6$.
- Also fixes reference $\varphi = 0$.