Zero Mode Counting in F-Theory via CAP

Martin Bies

String Pheno 2017

Overview

Task

4 dim. F-theory compactification $\Longrightarrow$ Count (anti)-chiral massless matter fields in 4d effective theory

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- Developed and implemented algorithms with M. Barakat et al. (https://github.com/homalg-project/CAP_project – 1003.1943, 1202.3337, 1210.1425, 1212.4068, 1409.2028, 1409.6100)

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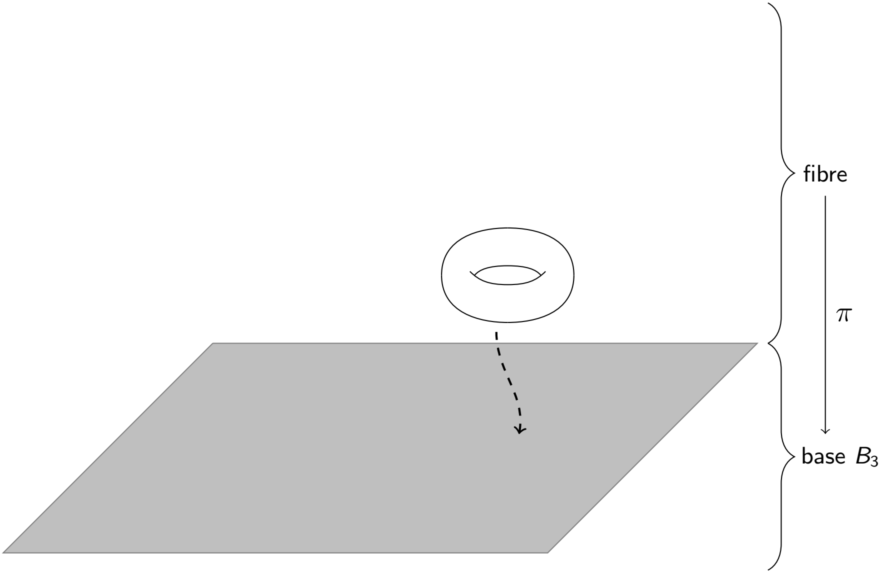

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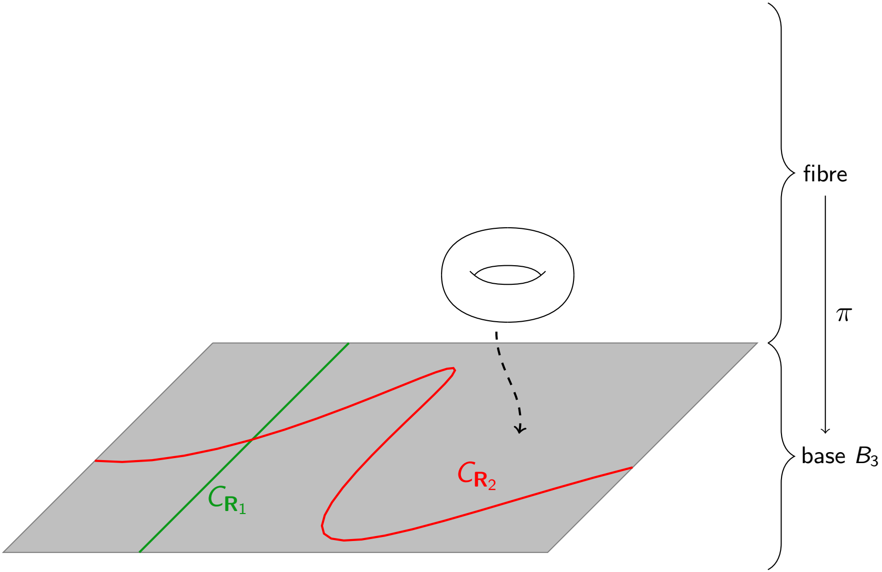

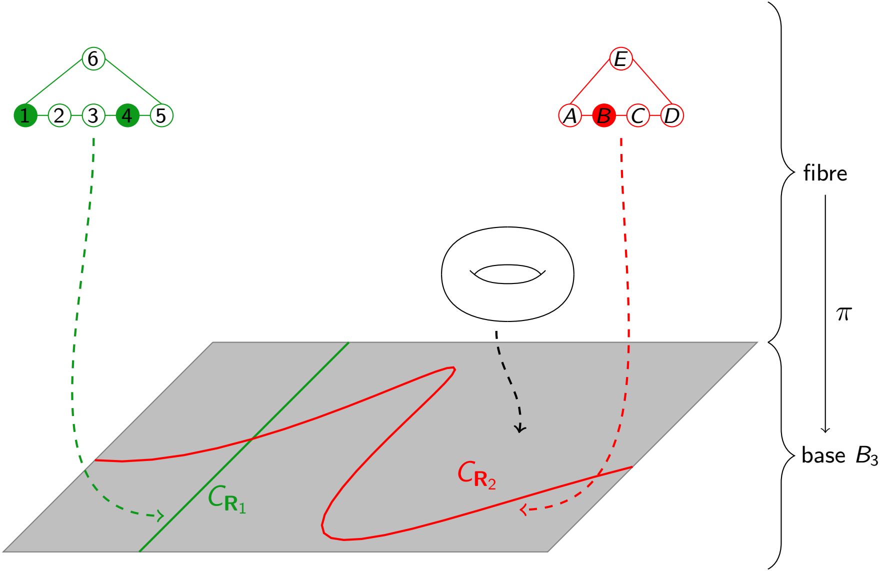
6
1
2
3
4
5
E
A
B
C
D
$C_{\mathbf{R}_1}$
$C_{\mathbf{R}_2}$
fibre
π
base B_3

Counting zero modes

From Physics of F-Theory to Line Bundles

$$\text{a-th state in rep. } \mathbf{R} \leftrightarrow \text{matter surface } S_{\mathbf{R}}^{a} = \sum_{i=1}^{n} a_i \mathbb{P}^1_i \left(C_{\mathbf{R}} \right)$$

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Zero Modes and Sheaf Cohomology

$$\text{chiral zero modes of } S^a_{\mathbf{R}} \leftrightarrow H^0 \left(C_{\mathbf{R}}, L \left(S^a_{\mathbf{R}}, A \right) \otimes \mathcal{O}_{\text{spin}, C_{\mathbf{R}}} \right)$$

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Why new algorithm?

$L(S^a_{\mathbf{R}}, A) = \mathcal{O}_{C_{\mathbf{R}}}(D)$. Extend $L(S^a_{\mathbf{R}}, A)$ by zero outside of $C_{\mathbf{R}}$.

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- to **all coherent sheaves** (i.e. not 'only' line bundles as *cohomCalg*)

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$\Rightarrow$ Look at exact sequence $\bigoplus_{j=1}^{R_2} S(e_j) \xrightarrow{A} \bigoplus_{i=1}^{R_1} S(d_i) \twoheadrightarrow M \to 0$,

which defines $M \in S\text{-}\mathrm{fpgrmod}$

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Answer

- $\widetilde{M} \cong \mathcal{O}_{X_\Sigma}(-D)$ via the sheafification functor

$$\widetilde{\ } : S\text{-fpgrmod} \to \mathfrak{Coh} X_\Sigma \, , \; N \mapsto \widetilde{N}$$

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$\Rightarrow$ Use $M \in S\text{-fpgrmod}$ as computer model for $\mathcal{O}_{X_\Sigma}(-D)$

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Input and Output

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- $M \in S\text{-fpgrmod}$

$\Longrightarrow$ $h^i\left(X_\Sigma, \widetilde{M}\right)$

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Step-by-step

1. Use *cohomCalg* to compute $(0 \leq k \leq \dim_{\mathbb{Q}} (X_\Sigma))$

$$V^k(X_\Sigma) := \left\{ L \in \mathrm{Pic}(X_\Sigma) \; , \; h^k(X_\Sigma, L) = 0 \right\}$$

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3. Compute $\mathbb{Q}$-dimension of $\mathrm{Ext}^i_S(I, M)_0$

$SU(5) \times U(1)$-Tate model from 1706.04616

Input and Output

- $C_{\mathbf{5}_{-2}} \subseteq \mathbb{P}^2_{\mathbb{Q}}$
- $L_{\mathbf{5}_{-2}} \leftrightarrow M$ and M defined by $S(-36) \oplus S(-39) \oplus S(-41) \oplus S(-23) \oplus S(-38) \to S(-6) \oplus S(-21) \twoheadrightarrow M \to 0$

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Apply Algorithm

1. Compute vanishing sets via *cohomCalg*:
$V^0(\mathbb{P}^2_{\mathbb{Q}}) = (-\infty, -1]_{\mathbb{Z}},\ V^1(\mathbb{P}^2_{\mathbb{Q}}) = \mathbb{Z},\ V^2(\mathbb{P}^2_{\mathbb{Q}}) = [-2, \infty)_{\mathbb{Z}}$

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2. Use vanishing sets to find ideal I (along idea of G. Smith): $I = B_{\Sigma}^{(44)} \equiv \langle x_0^{44}, x_1^{44}, x_2^{44} \rangle$

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2. $I = B_{\Sigma}^{(44)} \equiv \langle x_0^{44}, x_1^{44}, x_2^{44} \rangle$
3. Compute presentation of $\mathrm{Ext}^1_S\left(B_{\Sigma}^{(44)}, M\right)_0$:

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3. $\mathbb{Q}^{37425} \to \mathbb{Q}^{27201} \twoheadrightarrow \mathrm{Ext}^1_S\left(B_{\Sigma}^{(44)}, M\right)_0 \to 0$

$$\Rightarrow 28 = \dim_{\mathbb{Q}}\left[\mathrm{Ext}^1_S\left(B_{\Sigma}^{(44)}, M\right)_0\right] = h^1\left(\mathbb{P}^2_{\mathbb{Q}}, \widetilde{M}\right)$$

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$SU(5)xU(1)$-Tate Model from 1706.04616 ($\mathbf{R} = \mathbf{5}_{-2}$)

	$\widetilde{a_{1,0}}$	$\widetilde{a_{2,1}}$	$\widetilde{a_{3,2}}$	$\widetilde{a_{4,3}}$	$h^i\left(C_{\mathbf{R}}, L_{\mathbf{R}}\right)$
M_1	$(x_1 - x_2)^4$	x_1^7	x_2^{10}	x_3^{13}	$(22, 43)$
M_2	$(x_1 - x_2)\, x_3^3$	x_1^7	x_2^{10}	x_3^{13}	$(21, 42)$
M_3	x_3^4	x_1^7	$x_2^7\,(x_1 + x_2)^3$	$x_3^{12}\,(x_1 - x_2)$	$(11, 32)$
M_4	$(x_1 - x_2)^3\, x_3$	x_1^7	x_2^{10}	x_3^{13}	$(9, 30)$
M_5	x_3^4	x_1^7	$x_2^8\,(x_1 + x_2)^2$	$x_3^{11}\,(x_1 - x_2)^2$	$(7, 28)$
M_6	x_3^4	x_1^7	x_2^{10}	$x_3^8\,(x_1 - x_2)^5$	$(6, 27)$
M_7	x_3^4	x_1^7	$x_2^9\,(x_1 + x_2)$	$x_3^{10}\,(x_1 - x_2)^3$	$(5, 26)$

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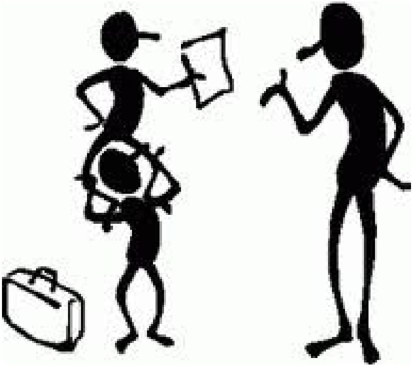
  - T-branes as coherent sheaves (Collinucci et al. 1410.4178)
  - ...

Thank you for your attention!



From Divisors to Modules

Input and Output

- $C = V(g_1, \ldots, g_k) \subseteq X_\Sigma$
- $D = V(f_1, \ldots, f_n) \in \mathrm{Div}(C)$

$\Longrightarrow$

M s.t. $\mathrm{supp}(\widetilde{M}) = C$ and $\widetilde{M}|_C \cong \mathcal{O}_C(-D)$

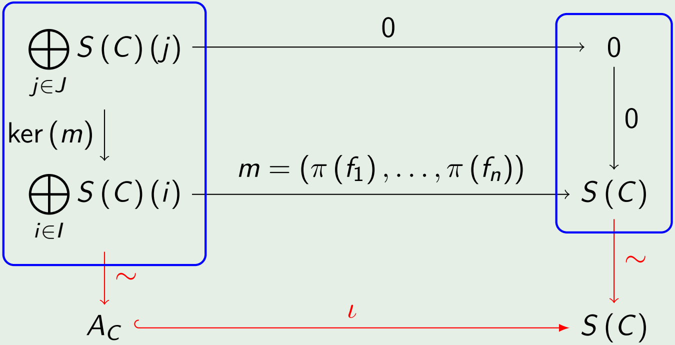
From Divisors to Modules

Input and Output

- $C = V(g_1, \ldots, g_k) \subseteq X_\Sigma$
- $D = V(f_1, \ldots, f_n) \in \mathrm{Div}(C)$

$\Longrightarrow$ M s.t. $\mathrm{supp}(\widetilde{M}) = C$ and $\widetilde{M}|_C \cong \mathcal{O}_C(-D)$

Step 1: $S(C) := S/\langle g_1, \ldots, g_k \rangle$, $\pi \colon S \twoheadrightarrow S(C)$



Step 2: Extend by zero to coherent sheaf on X_Σ

$$
\begin{array}{c}
\bigoplus_{j \in J} S(j) \\
\Big\downarrow {\scriptstyle \ker(m)'} \\
\bigoplus_{i \in I} S(i) \\
\Big\downarrow {\scriptstyle \sim} \\
A
\end{array}
\qquad \otimes \qquad
\begin{array}{c}
\bigoplus_{k \in K} S(k) \\
\Big\downarrow {\scriptstyle \begin{pmatrix} g_1 \\ \vdots \\ g_k \end{pmatrix}} \\
S(C) \\
\Big\downarrow {\scriptstyle \sim} \\
B
\end{array}
$$

$\Rightarrow M = A \otimes B$ satisfies $\mathrm{Supp}(\widetilde{M}) = C$ and $\widetilde{M}|_C \cong \mathcal{O}_C(-D)$

From Divisors to Modules III

Input and Output

- $C = V(g_1, \ldots, g_k) \subseteq X_\Sigma$
- $D = V(f_1, \ldots, f_n) \in \mathrm{Div}(C)$

$\Longrightarrow$

M s.t. $\mathrm{supp}(\widetilde{M}) = C$ and $\widetilde{M}|_C \cong \mathcal{O}_C(+D)$

Strategy

1. Compute A_C
2. Dualise via $A_C^\vee := \mathrm{Hom}_{S(C)}\left(S\left(C\right), A_C\right)$
3. Extend by zero by considering $A^\vee \otimes B$

$\Rightarrow$ $M^\vee := A^\vee \otimes B$ satisfies $\mathrm{Supp}(\widetilde{M}) = C$ and $\widetilde{M}|_C \cong \mathcal{O}_C\left(+D\right)$

An idea of the sheafification functor

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Affine open cover

- Toric variety X_Σ with Cox ring S

$\Rightarrow$ Covered by affine opens $\left\{ U_\sigma = \mathrm{Specm}(S_{(x^{\hat{\sigma}})}) \right\}_{\sigma \in \Sigma}$

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Localising ($\leftrightarrow$ restricting) a module

- $M \in S\text{-fpgrmod}$

$\Rightarrow$ $M_{(x^{\hat{\sigma}})}$ is f.p. $S_{(x^{\hat{\sigma}})}$-module

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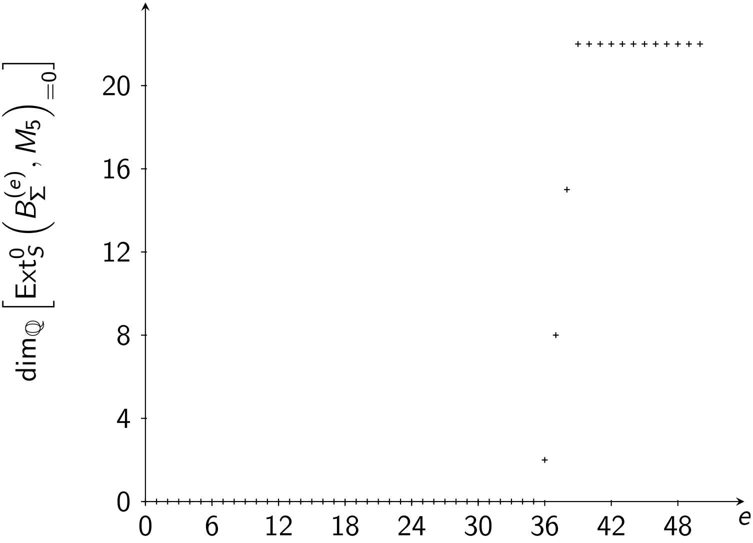
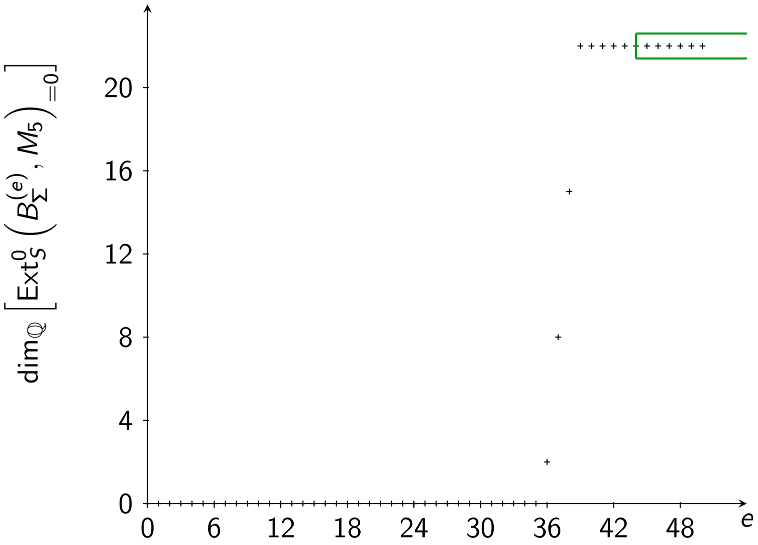
Consequence

- $M_{(x^{\hat{\sigma}})} \leftrightarrow$ coherent sheaf on $U_\sigma = \text{Specm}(S_{(x^{\hat{\sigma}})})$
- local sections: $\widetilde{M_{(x^{\hat{\sigma}})}}(D(f)) = M_{(x^{\hat{\sigma}})} \otimes_{S_{(x^{\hat{\sigma}})}} \left(S_{(x^{\hat{\sigma}})} \right)_f$

Module M_5 from 1706.04616: Quality Check I



Module M_5 from 1706.04616: Quality Check I



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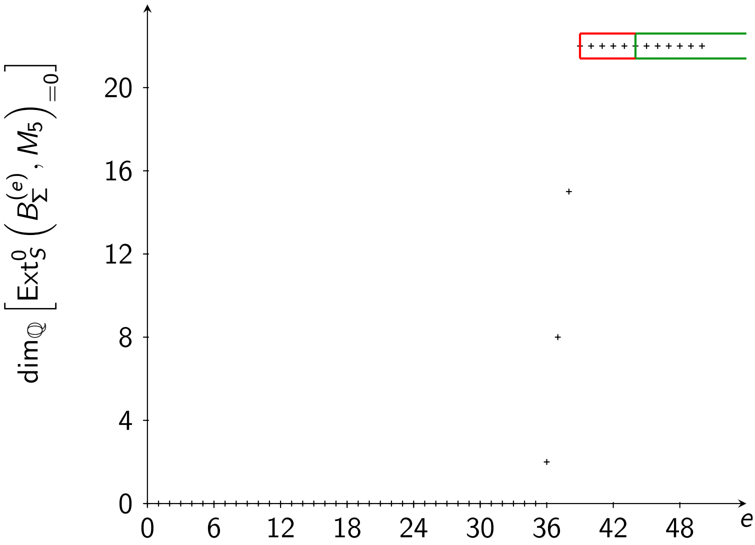
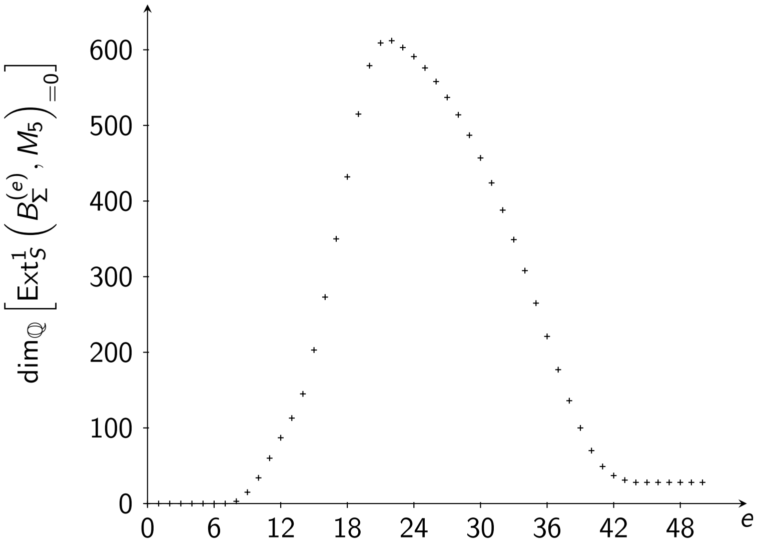
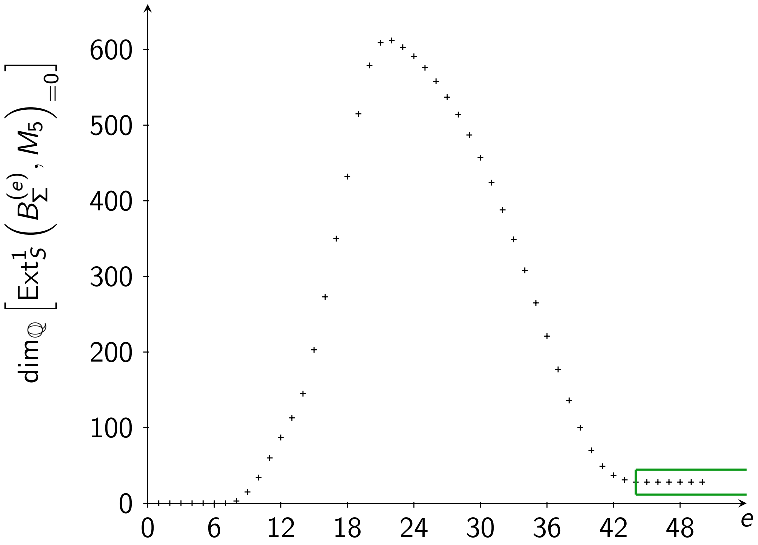


Module M_5 from 1706.04616: Quality Check II




$\dim_{\mathbb{Q}} \left[\mathrm{Ext}^1_S \left(B^{(e)}_{\Sigma}, M_5 \right)_{=0} \right]$
600
500
400
300
200
100
0
0
6
12
18
24
30
36
42
48
e

How to determine the ideal I in step 2 of algorithm?

Input

- $M \in S\text{-fpgrmod}$
- $V^k(X_\Sigma) = \left\{ L \in \operatorname{Pic}(X_\Sigma)\ ,\ h^k(X_\Sigma, L) = 0 \right\}$

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Preparation

- $p \in \mathrm{Cl}(X_\Sigma)$ **ample**, $m(p) = \{m_1, \ldots, m_k\}$ all monomials of degree p and $I(p, e) = \langle m_1^e, \ldots, m_k^e \rangle$
- Pick $e = 0$ and increase it until subsequent conditions are met

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How to find ideal I?

- Look at spectral sequence $\mathbb{E}_2^{p,q} \Rightarrow \mathrm{Ext}^{p+q}_{\mathcal{O}_{X_\Sigma}}\left(\widetilde{I(p,e)}, \widetilde{M} \right)$

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$\Rightarrow$ If no – increase e until this is the case!

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- Long exact sequence: sheaf cohomology $\leftrightarrow$ local cohomology

$\Rightarrow$ Increase e further until $H^m\left(\mathbf{C}^0\right) \cong \mathrm{Ext}^m_S(I(p,e), M)_0$