Homework 3 – Coding

Due: Thursday, February 3 – 10:00 am EST

Problem 1C (`Python`): Elementary row operations [20 Points]

In this exercise, $\mathcal{A}$ denotes a `numpy`-array.

1. Write a function *add_rows* with the following properties:
   - Input: $\mathcal{A}$, k, i, j.
   - Output: Numpy-array resulting from adding k times row j to row i.

   For $i = j$, rescale row i by $k + 1$.

2. Write a function *scale_row* with the following properties:
   - Input: $\mathcal{A}$, k, i.
   - Output: Numpy-array resulting from k times row i.

3. Write a function *switch_rows* with the following properties:
   - Input: $\mathcal{A}$, i, j.
   - Output: Numpy-array resulting from switching rows i and j.

4. Use these functions to compute the row reduced echelon form of C:

$$C = \begin{bmatrix} 0 & 2 & 3 & 5 \\ 5 & 6 & 7 & 13 \\ 9 & 10 & 11 & 21 \\ 13 & 14 & 15 & 29 \end{bmatrix} . \tag{1}$$

5. Let $i, j \in \mathbb{Z}_{>0}$ with $i > j$. Consider the elementary matrix $E_{ij}(k)$, whose non-trivial entries are 1's along the diagonal and k in row i column j. Write a function *elementary_matrix* with the following properties:
   - Input: k, i, j
   - Output: 4×4 numpy-array matching $E_{ij}(k)$.

6. Use the above functions to compute a PLU-factorization of A in problem 1T. Do this step-by-step by Gauss elimination. Print L, U and verify that $LU = A$.