Math 313/513, Spring 2021 **Martin Bies**

Homework 5 – Coding

Due: Thursday, February 24 – 10:00 am EST

Problem 1C: Transformation matrix [20 Points]

1. Write a `Python` function:
   Input: Two basis $\mathcal{B}_1$, $\mathcal{B}_2$ of $\mathbb{R}^n$. Output: $T_{\mathcal{B}_2\mathcal{B}_1}$.

2. Write a `Python` function:
   Input: Two basis $\mathcal{B}_1$, $\mathcal{B}_2$ of $\mathbb{R}^n$ and $A_{\mathcal{B}_1\mathcal{B}_1}$ Output: $A_{\mathcal{B}_2\mathcal{B}_2}$.

3. We will now identify a basis $\mathcal{B}'$ such that $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^3$ is easy:

$$A_{\mathcal{B}\mathcal{B}} = \begin{bmatrix} 3/2 & 1/2 & 0 \\ 1/2 & 3/2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \qquad \mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \tag{1}$$

   - Rotate the standard basis $\mathcal{B}$ by $\alpha \in I = \{0°, 1°, \ldots, 720°\}$ about the z-axis:

$$\mathcal{B}' = \{R_z\vec{e_1}, R_z\vec{e_2}, R_z\vec{e_3}\}, \quad R_z = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

     For all $\alpha \in I$, verify that $\mathcal{B}'$ is a basis.
     Hint: Use your function from homework 4.

   - Compute $A_{\mathcal{B}'\mathcal{B}'}$ with 5 digit precision for all angles $\alpha \in I$.
   - For which $\alpha \in I$ is $A_{\mathcal{B}'\mathcal{B}'}$, as computed by `Python`, approximately diagonal?