F-Theory Tools: String theory applications of OCSAR

Martin Bies

RPTU Kaiserslautern-Landau

Computeralgebra-Tagung 2023
Hannover, Germany
May 31, 2023

Motivation

- Questions to challenge string theory:
  - Does one of these solutions describe our universe?

    ($\leftrightarrow$ Holy grail of string phenomenology)
  - Can we make predictions beyond current experiments?

    ($\leftrightarrow$ Cool new technology?)
  - Can we understand the physical universe better?

    ($\leftrightarrow$ 19 experimentally determined parameters for particle physics – Really?!?)

Motivation

- Questions to challenge string theory:
  - Does one of these solutions describe our universe?
    ($\leftrightarrow$ Holy grail of string phenomenology)
  - Can we make predictions beyond current experiments?
    ($\leftrightarrow$ Cool new technology?)
  - Can we understand the physical universe better?
    ($\leftrightarrow$ 19 experimentally determined parameters for particle physics – Really?!?)
- My work: Vector-like spectra
  - Relatively recent with M. Cvetič, R. Donagi, L. Lin, M. Liu, M. Ong, F. Ruehle
    (2007.00009, 2102.10115, 2104.08297, 2205.00008, 2303.08144, and work in progress)
  - Many algorithmic aspects
    (toric varieties, intersection theory, categories, sheaf cohomologies, root bundles on nodal curves, . . .)

Motivation

- Questions to challenge string theory:
  - Does one of these solutions describe our universe?
    ($\leftrightarrow$ Holy grail of string phenomenology)
  - Can we make predictions beyond current experiments?
    ($\leftrightarrow$ Cool new technology?)
  - Can we understand the physical universe better?
    ($\leftrightarrow$ 19 experimentally determined parameters for particle physics – Really?!?)
- My work: Vector-like spectra
  - Relatively recent with M. Cvetič, R. Donagi, L. Lin, M. Liu, M. Ong, F. Ruehle
    (2007.00009, 2102.10115, 2104.08297, 2205.00008, 2303.08144, and work in progress)
  - Many algorithmic aspects
    (toric varieties, intersection theory, categories, sheaf cohomologies, root bundles on nodal curves, . . .)
- Most recent additions:
  - Toric geometry in OSCAR with L. Kastner. (2303.08110)
  - FTheoryTools in OSCAR with A. P. Turner, M. Zach, A. Frühbis-Krüger. (work in progress)

OSCAR: Open Source Computer Algebra Research system

- Goal: For computations in algebra, geometry, and number theory.

OSCAR: **O**pen **S**ource **C**omputer **A**lgebra **R**esearch system

- Goal: For computations in algebra, geometry, and number theory.
- Language: Julia.

OSCAR: **O**pen **S**ource **C**omputer **A**lgebra **R**esearch system

- Goal: For computations in algebra, geometry, and number theory.
- Language: Julia.
- Focus: Support complex computations which require a high level of integration of tools from different mathematical areas.

OSCAR: Open Source Computer Algebra Research system

- Goal: For computations in algebra, geometry, and number theory.
- Language: Julia.
- Focus: Support complex computations which require a high level of integration of tools from different mathematical areas.
- Website: https://www.oscar-system.org.

OSCAR: **O**pen **S**ource **C**omputer **A**lgebra **R**esearch system

- Goal: For computations in algebra, geometry, and number theory.
- Language: Julia.
- Focus: Support complex computations which require a high level of integration of tools from different mathematical areas.
- Website: https://www.oscar-system.org.
- **Integral** components: `GAP`, `Singular`, `Polymake`, `Antic` (`Hecke`, `Nemo`).

OSCAR: **O**pen **S**ource **C**omputer **A**lgebra **R**esearch system

- Goal: For computations in algebra, geometry, and number theory.
- Language: Julia.
- Focus: Support complex computations which require a high level of integration of tools from different mathematical areas.
- Website: https://www.oscar-system.org.
- **Integral** components: `GAP`, `Singular`, `Polymake`, `Antic` (`Hecke`, `Nemo`).
- Examples:
  - Toric geometry: Polyhedral geometry, algebraic geometry, combinatorics + . . .
  - `FTheoryTools`: Toric and algebraic geometry, intersection theory, Lie groups, . . .

OSCAR: Open Source Computer Algebra Research system

- Goal: For computations in algebra, geometry, and number theory.
- Language: Julia.
- Focus: Support complex computations which require a high level of integration of tools from different mathematical areas.
- Website: https://www.oscar-system.org.
- **Integral** components: GAP, Singular, Polymake, Antic (Hecke, Nemo).
- Examples:
  - Toric geometry: Polyhedral geometry, algebraic geometry, combinatorics + ...
  - FTheoryTools: Toric and algebraic geometry, intersection theory, Lie groups, ...
- More information e.g. in latest *Computeralgebra-Rundbrief*:
  - M. Horn: OSCAR: An introduction
  - M. Bies, L. Kastner: Toric geometry in OSCAR. (2303.08110)

Toric Geometry in OSCAR

- Documentation: https://docs.oscar-system.org/stable/.
- Tutorials: https://www.oscar-system.org/tutorials/.

Toric Geometry in OSCAR

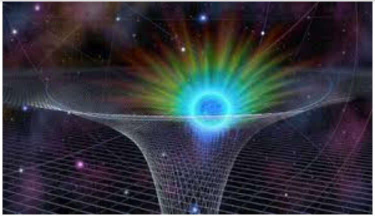
- Documentation: https://docs.oscar-system.org/stable/.
- Tutorials: https://www.oscar-system.org/tutorials/.
- Used for instance by F. Witt and L. Remke.
  $\rightarrow$ Improved/extended tutorial to appear shortly ...

Toric Geometry in OSCAR

- Documentation: https://docs.oscar-system.org/stable/.
- Tutorials: https://www.oscar-system.org/tutorials/.
- Used for instance by F. Witt and L. Remke.
  → Improved/extended tutorial to appear shortly . . .
- Check it out!

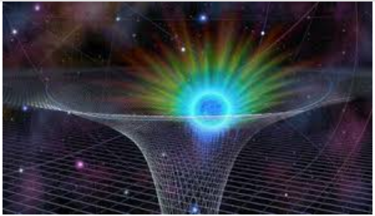
Questions so far?

String theory = General relativity + Standard Model?

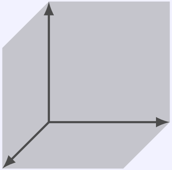


+

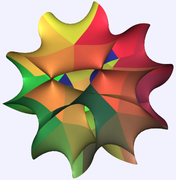
String theory = General relativity + Standard Model?



$+$



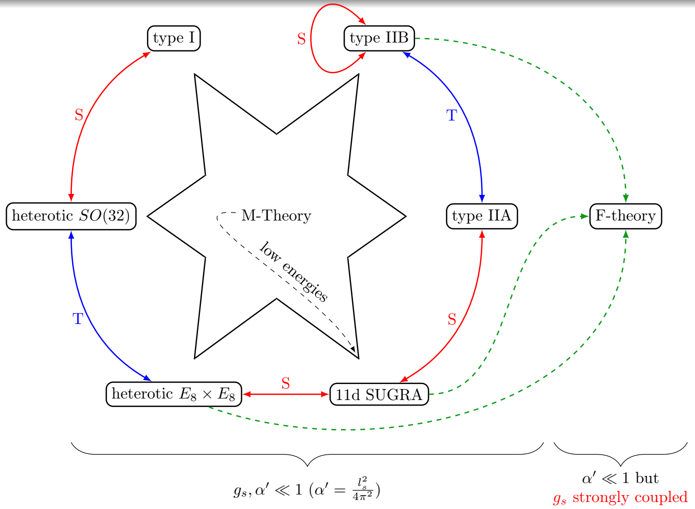
$\times$



4-dim. world $\mathcal{W}$ ‘small‘ 6 real-dim. manifold $\mathcal{B}_3$

Challenge: Find $\mathcal{B}_3$ s.t. ST reproduces 4d physics.

Different types of String theory



Singular elliptic fibrations for F-theory

Recent review: [Weigand '18]

- Particle physics deals with "fields".
- Fields are, loosely speaking, functions. (More precisely: Sections of vector bundles.)
- F-theory: Axio-dilaton field τ is super important:
  - $\tau\colon B_3 \to \mathbb{C}\,,\ p \mapsto C_0(p) + \frac{i}{g_s(p)}$.
  - Physics invariant under $SL(2,\mathbb{Z})$ transformation of τ:

$$\tau \mapsto \frac{a\tau + b}{c\tau + d}\,, \quad a, b, c, d \in \mathbb{Z} \text{ and } ad - bc = 1\,.$$

    $\Rightarrow$ Axio-dilation τ is complex structure modulus of elliptic curve $\mathbb{C}_{1,\tau}$.
  - $\tau(p) \to i\infty$ when $g_s(p) \to 0$ (Physics lingo: Vicinity of D7-branes.)
    $\Rightarrow \mathbb{C}_{1,\tau}$ becomes singular.

Singular elliptic fibrations for F-theory

Recent review: [Weigand '18]

- Particle physics deals with "fields".
- Fields are, loosely speaking, functions. (More precisely: Sections of vector bundles.)
- F-theory: Axio-dilaton field τ is super important:
  - $\tau \colon B_3 \to \mathbb{C}\,,\; p \mapsto C_0(p) + \frac{i}{g_s(p)}$.
  - Physics invariant under $SL(2, \mathbb{Z})$ transformation of τ:

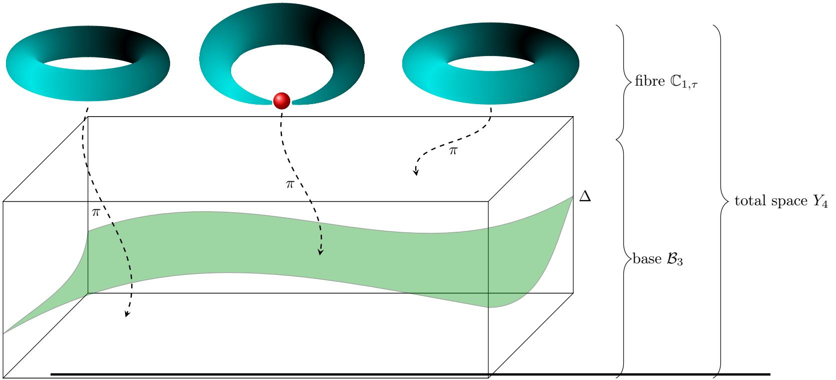
$$\tau \mapsto \frac{a\tau + b}{c\tau + d}\,, \quad a, b, c, d \in \mathbb{Z} \text{ and } ad - bc = 1\,.$$

    $\Rightarrow$ Axio-dilation τ is complex structure modulus of elliptic curve $\mathbb{C}_{1,\tau}$.
  - $\tau(p) \to i\infty$ when $g_s(p) \to 0$ (Physics lingo: Vicinity of D7-branes.)
    $\Rightarrow$ $\mathbb{C}_{1,\tau}$ becomes singular.

Consequence [Vafa '96], [Morrison Vafa '96]

Singular elliptic fibration as book-keeping device/consistency check of F-theory.

Cartoon: Singular elliptic fibrations for F-theory



IIB-SUGRA	Geometry
union of loci of D7-branes in IIB-compactification	Singular locus Δ of elliptic fibration $\mathbb{C}_{1,\tau} \hookrightarrow Y_4 \overset{\pi}{\twoheadrightarrow} \mathcal{B}_3$

Singularities meet F-theory

Singularities meet F-theory

- Strategy 1: Do not resolve the singularities
  Hard to extract the physics, but some attempts do exist.

  [Anderson Heckman Katz '13], [Collinucci Savelli '14], [Collinucci Giacomelli Savelli Valandro '16]

Singularities meet F-theory

- Strategy 1: Do not resolve the singularities
  Hard to extract the physics, but some attempts do exist.
  [Anderson Heckman Katz '13], [Collinucci Savelli '14], [Collinucci Giacomelli Savelli Valandro '16]
- Strategy 2: Resolve the singularities ($\leftrightarrow$ Coulomb branch of dual 3d M-theory)
  - For (simple) physics interpretation, must resolve **crepantly**.
  - Employ (weighted) blowup sequence. . . . [Arena Jefferson Obinna '23]

  $\Rightarrow$ Challenges to find a crepant resolution:
    - Q-factorial terminal singularities cannot be resolved crepantly.
    - Currently, no algorithm for crepant (weighted blowup) resolution.
  - Sometimes find non-flat fibrations: Physics not clear.
    [Lawrie Schafer-Nameki '12], [Apruzzi Heckman Morrison Tizzano '18], ...

Singularities meet F-theory

- Strategy 1: Do not resolve the singularities
  Hard to extract the physics, but some attempts do exist.
  [Anderson Heckman Katz '13], [Collinucci Savelli '14], [Collinucci Giacomelli Savelli Valandro '16]
- Strategy 2: Resolve the singularities ($\leftrightarrow$ Coulomb branch of dual 3d M-theory)
  - For (simple) physics interpretation, must resolve **crepantly**.
  - Employ (weighted) blowup sequence. ... [Arena Jefferson Obinna '23]

  $\Rightarrow$ Challenges to find a crepant resolution:
    - Q-factorial terminal singularities cannot be resolved crepantly.
    - Currently, no algorithm for crepant (weighted blowup) resolution.
  - Sometimes find non-flat fibrations: Physics not clear.
    [Lawrie Schafer-Nameki '12], [Apruzzi Heckman Morrison Tizzano '18], ...
- One goal of `FTheoryTools`: Automate strategy 2.

Goals for FTheoryTools

1. Find and implement algorithm for **crepant** resolution:
   - Many details known in F-theory literature.
   - Crepant is “exotic“ condition in mathematics.

   ⇒ No algorithm known yet, but we can try . . .

Goals for FTheoryTools

1. Find and implement algorithm for **crepant** resolution:
   - Many details known in F-theory literature.
   - Crepant is "exotic" condition in mathematics.
   - ⇒ No algorithm known yet, but we can try ...
2. Generalize/implement techniques:
   - A lot of toric functionality in OSCAR [Bies Kastner '23]
   - Many interesting techniques known [Jefferson Taylor Turner '21], [Jefferson Turner '22], ...
   - Sometimes, we need/wish to go beyond the toric regime (e.g. non-toric (crepant) blowup).

Goals for FTheoryTools

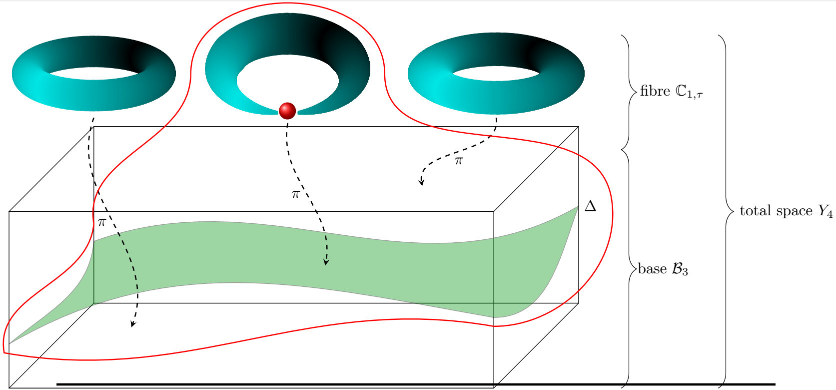
1. Find and implement algorithm for **crepant** resolution:
    - Many details known in F-theory literature.
    - Crepant is “exotic“ condition in mathematics.

   ⇒ No algorithm known yet, but we can try . . .
2. Generalize/implement techniques:
    - A lot of toric functionality in OSCAR [Bies Kastner ‘23]
    - Many interesting techniques known [Jefferson Taylor Turner ‘21], [Jefferson Turner ‘22], . . .
    - Sometimes, we need/wish to go beyond the toric regime (e.g. non-toric (crepant) blowup).
3. Many models studied in large detail in F-theory literature:
    - Resolutions, topological data, . . . known.
    - Study same model with different techniques.

   ⇒ `LiteratureModels`

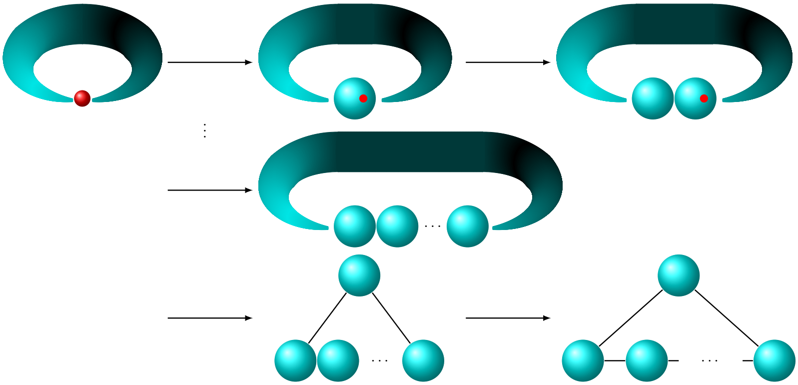
Questions so far?

Cartoon: Singular elliptic fibration for F-theory



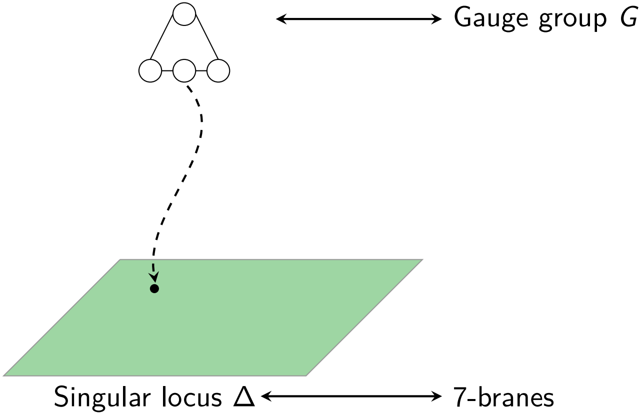
IIB-SUGRA	Geometry
union of loci of D7-branes in IIB-compactification	Singular locus Δ of elliptic fibration $\mathbb{C}_{1,\tau} \hookrightarrow Y_4 \overset{\pi}{\twoheadrightarrow} \mathcal{B}_3$

Cartoon of blow-up resolution



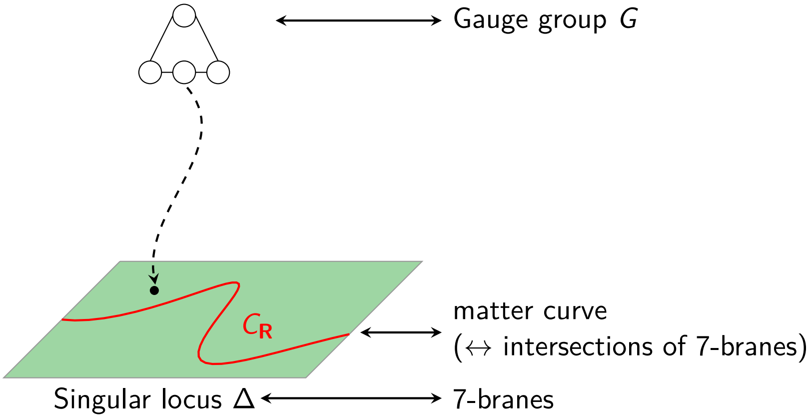
Massless matter

[Katz Vafa '96], [Witten '96], [Grassi,Morrison '00 & '11], [Morrison,Taylor '11], [Grassi,Halverson,Shaneson '13], [Cvetič,Klevers,Piragua,Taylor '15], [Anderson,Gray,Raghuram,Taylor '15], [Klevers,Taylor '16], [Klevers,Morrison,Raghuram,Taylor '17], . . .



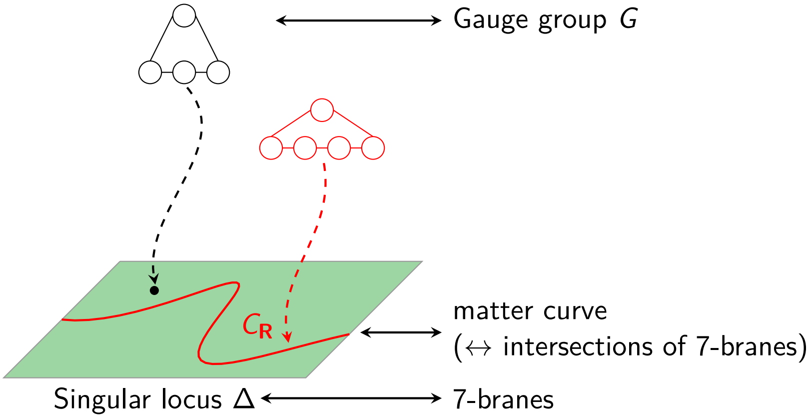
Massless matter

[Katz Vafa '96], [Witten '96], [Grassi,Morrison '00 & '11], [Morrison,Taylor '11], [Grassi,Halverson,Shaneson '13], [Cvetič,Klevers,Piragua,Taylor '15], [Anderson,Gray,Raghuram,Taylor '15], [Klevers,Taylor '16], [Klevers,Morrison,Raghuram,Taylor '17], ...



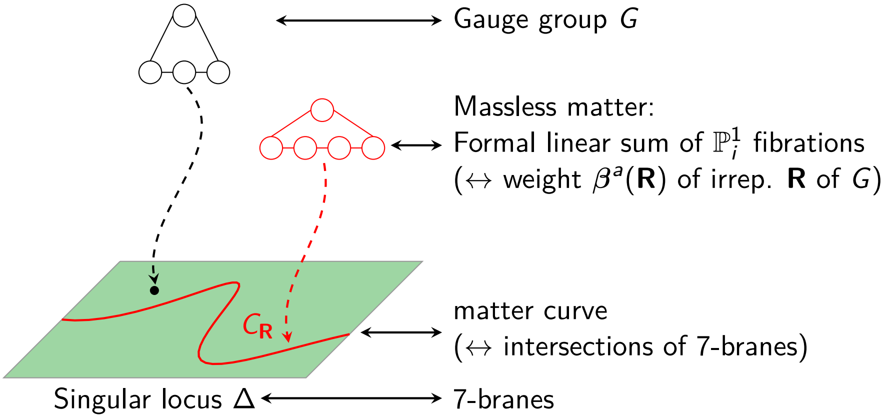
Massless matter

[Katz Vafa '96], [Witten '96], [Grassi,Morrison '00 & '11], [Morrison,Taylor '11], [Grassi,Halverson,Shaneson '13], [Cvetič,Klevers,Piragua,Taylor '15], [Anderson,Gray,Raghuram,Taylor '15], [Klevers,Taylor '16], [Klevers,Morrison,Raghuram,Taylor '17], ...

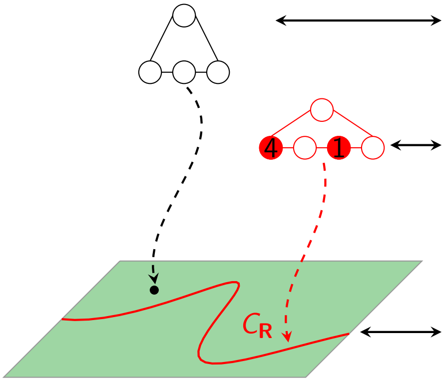


Massless matter

[Katz Vafa '96], [Witten '96], [Grassi,Morrison '00 & '11], [Morrison,Taylor '11], [Grassi,Halverson,Shaneson '13], [Cvetič,Klevers,Piragua,Taylor '15], [Anderson,Gray,Raghuram,Taylor '15], [Klevers,Taylor '16], [Klevers,Morrison,Raghuram,Taylor '17], . . .



Massless matter [Katz Vafa '96], [Witten '96], [Grassi,Morrison '00 & '11], [Morrison,Taylor '11], [Grassi,Halverson,Shaneson '13], [Cvetič,Klevers,Piragua,Taylor '15], [Anderson,Gray,Raghuram,Taylor '15], [Klevers,Taylor '16], [Klevers,Morrison,Raghuram,Taylor '17], ...



Gauge group G

Massless matter:
Formal linear sum of $\mathbb{P}^1_i$ fibrations
($\leftrightarrow$ weight $\beta^a(\mathbf{R})$ of irrep. $\mathbf{R}$ of G)

matter curve
($\leftrightarrow$ intersections of 7-branes)

Singular locus Δ $\longleftrightarrow$ 7-branes

An F-theory global Tate model More details: [Weigand '18]

- Consider $\mathbb{P}^{2,3,1}$ with coordinates $[x : y : z]$.

(In analogy to "ordinary" projective space: $(\lambda^2 x, \lambda^3 y, \lambda z) \sim (x, y, z)$ and $x = y = z = 0$ is forbidden.)

- Let B_3 be a complete, Kaehler 3-fold s.t. there exist

$$0 \neq a_i \in H^0\left(B_3, \overline{K}_{B_3}^{\otimes i}\right), \qquad i \in \{1, 2, 3, 4, 6\}.$$

- Define the Tate polynomial ("long Weierstrass equation"):

$$P_T = y^2 + a_1 xyz + a_3 yz^3 - x^3 - a_2 x^2 z^2 - a_4 xz^4 - a_6 z^6 .$$

- Fix $p \in B_3$. Then $V(P_T(p)) \subset \mathbb{P}^{2,3,1}$ is a torus surface.

$\Rightarrow$ Elliptic fibration $\pi\colon Y_4 \twoheadrightarrow B_3$ with section $[x : y : z] = [1 : 1 : 0]$.

("Global": P_T defines the model for every $p \in B_3$.)

Global Tate model to Weierstrass model More details: [Weigand '18]

- Consider global Tate model defined by $a_i \in H^0\left(B_3, \overline{K}_{B_3}^{\otimes i}\right)$ and

$$P_T = y^2 + a_1 xyz + a_3 yz^3 - x^3 - a_2 x^2 z^2 - a_4 xz^4 - a_6 z^6 \,.$$

- We define a few quantities:

$$b_2 = 4a_2 + a_1^2 \,, \quad b_4 = 2a_4 + a_1 a_3 \,, \quad b_6 = 4a_6 + a_3^2 \,,$$
$$f = -\frac{1}{48}\left(b_2^2 - 24 b_4\right) \,, \quad g = \frac{1}{864}\left(b_2^3 - 36 b_2 b_4 + 216 b_6\right) \,.$$

⇒ (Short) Weierstrass model defined by f, g and

$$P_W = y^2 - x^3 - fxz^4 - gz^6 \,.$$

The singular loci of the Tate/Weierstrass model are

$$V(\Delta) = V(4f^3 + 27g^2) \subset B_3 \,.$$

An $SU(5) \times U(1)$ F-theory global Tate model

Fine tune F-theory global Tate model

Wish to have particular singularity over hypersurface $V(w) \subset B_3$.

One particular model [Krause Mayrhofer Weigand '11]

- Assume that B_3 allows us to factor the sections a_i:

$$a_1 = a_1\,,\;\; a_2 = a_{2,1} w\,,\;\; a_3 = a_{3,2} w^2\,,\;\; a_4 = a_{4,3} w^3\,,\;\; a_6 \equiv 0\,.$$

$\Rightarrow$ $\Delta = 4f^3 + 27g^2 = w^5 \cdot P$, with complicated polynomial P.

- Singularities:
  - $\mathrm{ord}_{V(w)}(f, g, \Delta) = (0, 0, 5)$: I_5-singularity $\leftrightarrow$ $SU(5)$
  - $\mathrm{ord}_{V(P)}(f, g, \Delta) = (0, 0, 1)$: I_1-singularity $\leftrightarrow$ "Not relevant"

$U(1)$ from Mordell-Weil group of elliptic fibration ...

(More information: Kodaira classification, Tate table, Weierstrass table)

Resolution for $SU(5) \times U(1)$ F-theory global Tate model

- Blowup sequence worked out in literature [Krause Mayrhofer Weigand '11]:

$$
\begin{aligned}
(x, y, w) &\to (xe_1, ye_1, we_1)\,, \\
(y, e_1) &\to (ye_4, e_1 e_4)\,, \\
(x, e_4) &\to (xe_2, e_4 e_2)\,, \\
(y, e_2) &\to (ye_3, e_2 e_3)\,, \\
(x, y) &\to (xs, ys)\,.
\end{aligned}
$$

Resolution for $SU(5) \times U(1)$ F-theory global Tate model

- Blowup sequence worked out in literature [Krause Mayrhofer Weigand '11]:

$$
\begin{aligned}
(x, y, w) &\to (xe_1, ye_1, we_1)\,,\\
(y, e_1) &\to (ye_4, e_1 e_4)\,,\\
(x, e_4) &\to (xe_2, e_4 e_2)\,,\\
(y, e_2) &\to (ye_3, e_2 e_3)\,,\\
(x, y) &\to (xs, ys)\,.
\end{aligned}
$$

- Demonstrate with **experimental stage of** `FTheoryTools`: https://docs.oscar-system.org/dev/Experimental/FTheoryTools/tate/

Outlook and status of FTheoryTools

- Status:
  - Experimental stage of OSCAR. (https://www.oscar-system.org/)
  - Documentation: https://docs.oscar-system.org/stable/Experimental/FTheoryTools/introduction/.
  - Tutorial: https://www.oscar-system.org/tutorials/.

Outlook and status of FTheoryTools

- Status:
  - Experimental stage of OSCAR. (https://www.oscar-system.org/)
  - Documentation: https://docs.oscar-system.org/stable/Experimental/FTheoryTools/introduction/.
  - Tutorial: https://www.oscar-system.org/tutorials/.
- Goals:
  - Support (all) literature models.
  - Support (all) known techniques. ..., [Grimm Hayashi '11], [Krause Mayrhofer Weigand '11], [Braun Grimm Keitel '13], [Cvetič Grassi Klevers Piragua '14], [Cvetič Klevers Peña Oehlmann Reuter, '15], [Lin Mayrhofer Till Weigand '16], [Lin Weigand '16], [Jefferson Taylor Turner '21], [Jefferson Turner '22], [Arena Jefferson Obinna '23], ...
  - Seek and implement algorithm for **crepant** resolution.
  - Compute refined data of the resolved space. (E.g. towards vector-like spectra).

Outlook and status of FTheoryTools

- Status:
  - Experimental stage of OSCAR. (https://www.oscar-system.org/)
  - Documentation: https://docs.oscar-system.org/stable/Experimental/FTheoryTools/introduction/.
  - Tutorial: https://www.oscar-system.org/tutorials/.
- Goals:
  - Support (all) literature models.
  - Support (all) known techniques. ..., [Grimm Hayashi '11], [Krause Mayrhofer Weigand '11], [Braun Grimm Keitel '13], [Cvetič Grassi Klevers Piragua '14], [Cvetič Klevers Peña Oehlmann Reuter, '15], [Lin Mayrhofer Till Weigand '16], [Lin Weigand '16], [Jefferson Taylor Turner '21], [Jefferson Turner '22], [Arena Jefferson Obinna '23], ...
  - Seek and implement algorithm for **crepant** resolution.
  - Compute refined data of the resolved space. (E.g. towards vector-like spectra).

⇒ Opportunity: Testing ground for new techniques and finding new physics.

Thank you for your attention!